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ELECTROMAGNETIC LAUNCH TECHNOLOGY ASSESSMENT

Scientific Basis and Unified Treatment: Forces and Electromechanical Power Conversion (Analytical and Numerical Methods)

June 1990

by

M. D. Driga

to

BATTELLE
Tactical Technology Center
505 King Avenue
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This report suggests that one of the reasons why the electromagnetic (EM) hypervelocity launchers and associated power supply technologies have not reached their expectations is that most research has focused on simple railgun accelerators and homopolar generators. Complex rotating flux compressors would drastically improve the performance of EM launchers. This report explores the computations for the design of such complex magnetic flux compressors and heteropolar hypervelocity accelerators. It presents a finite element method formulation for electromagnetic field diffusion.

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PREFACE

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DISCLAIMER

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CHAPTER 1

INTRODUCTION

1.1 A NEED FOR A NEW APPROACH TO ELECTROMAGNETIC HYPERVELOCITY LAUNCHERS AND THEIR POWER SUPPLIES

Electromagnetic propulsion, in the sense of hypervelocity launchers has been hailed at the beginning of this decade as "an emerging technology with the potential of having a major impact in every aspect of our lives."[1]

Contrary to the general expectations, however, the initial hopes have not materialized yet, the projectile velocity records remaining essentially unbroken: for railguns 5.7 km/sec obtained by Richard Marshal and coworkers at the Australian National University at Canberra (1970) [2], and for coilguns, 4.9 km/sec obtained by V.N. Bondoletov and coworkers in 1975 in the Soviet Union.[3]

A relative improvement has occurred in the kinetic energy stored in the moving member of the electromagnetic accelerator [4,5], but at markedly lower velocities. If this is the direction of improvement, then energies stored in the electric trains built with linear electric motors in Japan and Western Europe will surpass any such records.

One of the reasons why the current state of the art in electromagnetic launchers and associated power supplies technology has not reached the expectations is the fact that most of the activity in this domain has been focused on the easiest solutions of simple railgun accelerators (requiring the

fabrication technology available in virtually any laboratory or shop) which were manufactured predominantly by laboratories which had power supplies already built for other programs (homopolar generators in conjunction with an inductor and an opening switch, or high performance capacitors).

The choice for homopolar generators is also questionable since they are known for their low voltage and low power density. A recent study (under auspices of D.C. Hardison Panel) has shown that homopolar machines are not well suited for electromagnetic launching applications. (Elliott [6]). The actual power supply consists not only of the homopolar machines, but includes heavy and voluminous inductors in which magnetic energy is transiently stored and explosive opening switches which actually provide the high voltage necessary in order to drive high currents into railguns (though at a very low efficiency compared to alternative capacitor schemes). For Richard Marshall and coworkers at Canberra [2], the homopolar generator was existent and the only choice.

A smaller effort was directed toward heteropolar structures used as accelerators in the form of induction coilguns and synchronous commutated coilguns. Concepts and preliminary designs have been performed for such alternative structures, but no notable experiment has been built or tested yet. The only ones tested have exhibited less than mediocre results [7].

Thus, there is a need for a scientific look, from a fundamental point of view, at the general problem of electromagnetic accelerators and their power supplies.

The analogy with the electrical machine counterparts - rotating or even linear - has multiple limitations and can be counterproductive or even misleading, due to the highly transient and short duration operation of accelerators as compared with the mostly steady-state performance of regular electrical machines.

a. The first basic feature of all electromagnetic launchers is the requirement of complete integration with their power supplies. At each moment of operation, the stator barrel of the accelerator, the projectile, and the power supply must form a carefully tuned system, capable of achieving a uniform and efficient electromechanical energy conversion with no contacts or feedback control. The tuning of the system varies during the launch since the parameters of the system must change in order to achieve an almost constant acceleration and uniformly distributed body forces and heating in the armature as well as in the barrel as a means of surviving the mechanical and thermal stresses [8].

b. Another basic feature of electromagnetic launchers and their power supplies is the variability of their structure [9]. The classification patterns and the characterization following a certain classification in the domain of classical electrical machines do not translate identically to the transient nature of accelerators. For example, the synchronous and asynchronous regimes trespass into each other during operation and the

customary methods of analysis for one or another regime cannot be used; in commutated launchers, each commutation occurs at a different velocity and position and at different values of the currents flowing inside of the projectile and, thus, all the classical patterns of analytical treatment of electromechanical devices become obsolete and not directly applicable.

c. The phenomena of flux compression existing only in some transient regime in classical electrical machines play a fundamental role in both launchers and electromechanical power supplies of high performance and efficiency.

d. In order to completely define and characterize the highly transient operation of both accelerator and power supply, finite element, three-dimensional, transient, electromagnetic computer codes are necessary. The survival of the projectile, the compactness of the launcher, and complex problems of shielding of the electronics in the payload for "smart" projectile require the exact knowledge in time and space, at any point in the launcher barrel, at any point in the projectile, and at any relative position between them of the electromagnetic fields. From there thermal fields and ensuing mechanical stresses can be calculated readily transferring data to similar thermal and stress finite element codes [10].

What is the most efficient path toward hypervelocity electromagnetic launchers and what are the power supplies which best conform to such

accelerators? In order to answer this question we will concentrate on the "promising" heteropolar structures and will relegate to other treatments and publications about generalized railguns in which some high impedance variants may bring some hope of improvement in the performance of homopolar structures used as launchers. (IEEE Transactions on Plasma Science No 3, June 1989, is devoted almost exclusively to railguns.)

The Bondoletov experiment, obtaining 4.9 km/sec in one single stage, shows the efficiency of repulsion coils, in attaining hypervelocities, for macroparticle. (This reminds us of the fact that, in the realm of conventional electrical machines, the repulsion induction motors are capable of high velocities, in very efficient schemes.)

There are several definite ideas as points of reference on the most efficient path toward hypervelocity electromagnetic launchers.

a. The first idea is a natural thought derived from the experiment [3] mentioned above: to increase the number of stages from one to several, obtaining a "Generalized Bondoletov Accelerator" (GBA).

b. In order to obtain a uniform force profile in the projectile and the barrel, a current must be impressed in the projectile before the acceleration had begun [11]. A liquid nitrogen cooled projectile has the potential of maintaining the persistent current during the short time of the launching. At liquid nitrogen

temperature, the specific heat has still an important value, while the resistance of the copper decreases more than six times.

c. Another fundamental characteristic of electromagnetic hypervelocity accelerators is not only the fact that the space around the projectile where the electromechanical power conversion takes place,is strictly localized to a small traction of the total length of the accelerator, but, also, that the parameters of such conversion are continuously changing from the breech toward the muzzle of the launcher.

Then, using different power supplies for different segments of the launcher stator (barrel) in the moments when the projectile is contained in the respective segment (Driga [11]) improves drastically the efficiency of the system. Discrete coils in the GBA are pulsed by dedicated electrical machines at the proper intervals of time.

d. It is shown by Driga, et. al in [8] and [12] that the traveling wave for the induction or synchronous launchers must accelerate at a determined pace with respect to the projectile. It is preferable to obtain such an accelerated wave by a continuous increase in the frequency of the power supply, rather than modifying the polar pitch of the winding. When pulses of voltage and current are used for discrete coils, for example, this requirement is modified into decreasing the pulse width and increasing its amplitude as the projectile is accelerated.

The process can also be described a generalized rising frequency of the power supply. Our notion of frequency variation (learned from the customary electrical machines, e.g. the starting of synchronous motors using a varying frequency power supply with many cycles) must be modified for the case when only one cycle, or even less, is present. A generalization in this respect means that the shape of the voltage and current must vary parabolically -the parameters of the parabola being determined by the acceleration values and patterns - at the given individual stator coil (Driga[13]).

CHAPTER 2

ANALYTICAL AND NUMERICAL TREATMENT OF THE ELECTROMAGNETIC LAUNCHERS AND THEIR POWER SUPPLIES INCORPORATING MAGNETIC FLUX COMPRESSION

2.1 A DISCUSSION ON COMPENSATION AND FLUX COMPRESSION

In the recent development of electromagnetic launch technology, electromagnetic launchers of higher impedance (coilguns - as heteropolar converters, or multiturn, augmented railguns) have not achieved the same advances as railgun launchers. There are several reasons for this. One is the complexity of such high impedance devices, where the payoffs of high efficiency, high performance, and, especially, flexibility are counterbalanced by extreme demands in computations and design, requiring sophisticated transient 3D electromagnetic computer codes series connected, or interacting, with 3D stress and thermal finite element codes. These codes require high-performance supercomputers and the personnel which few laboratories have been able to dedicate to this task.

A second, even more demanding reason, is the need for dedicated, high-voltage power supplies with capabilities for complex pulse tailoring, advancing considerably the technology of electromechanical pulsed power converters.

Recent studies (Elliott [6]) have shown that the inherently low-voltage homopolar machines are not well suited for many electromagnetic launching

applications. The actual power supply consists not only of the homopolar machine, but includes heavy and voluminous inductors in which magnetic energy is stored by discharge of the homopolar machine and explosive opening switches which actually provide the high voltage necessary in order to drive high currents into railguns (though at a very low efficiency, compared to alternative capacitor schemes).

This combination of low-voltage rotating machines, matching inductors, and opening switches cannot satisfy the waveshape complexity required by the higher impedance coilguns, even in a condition of extensive modularity.

A step ahead is represented by the Kaman alternator [6] which, being a higher voltage machine, leads to considerable improvement of efficiency when used in a similar scheme, due to a more rapid charging of the inductive store.

High-voltage capacitors achieve higher efficiency and recovery of an important part of the discharge energy. Their drawback lies in the fact that the limits of energy storage densities in electric fields are orders of magnitude lower than those for energy densities possible in magnetic fields. Also, they require elaborate power conditioning schemes, even for simple railguns. Complex power conditioning systems for the complex waveshapes required by heteropolar launchers such as coilguns will have an adverse impact to the efficiency of the of the capacitor schemes.

Compensated pulsed alternators have the advantages of high voltage, high efficiency, and capability of intermediate storage of energy in magnetic fields. They are frequently described as a one-element power supply (Figure 1), or single-element power supply philosophy, in papers by Driga [8] and Driga, et al. [13], replacing and modifying complex multielement energy conversion, storage, and power conditioning.

However, the devices described above are not truly compensated alternators since both general procedures for obtaining the complex pulse shaping by topological distributions (mainly harmonic synthesis) and by dynamic interactions (mainly by nonuniformly distributed shields and other types of compensation, by change of alignment of different machines windings, as well as the use of a pulse transformer incorporated in machine), represents a means to decompensate, to undo the machine compensation. The descriptions of these machines includes flux compression elements which supposedly provide "active compensation." Actually, flux compression is not compensation, as will be shown below. However, rotating flux compressors have all the promising characteristics to become the primary choice for power supplies for high power, complex heteropolar launchers (complex coilguns) for military, space, and industrial applications.

The computational efforts for the design of rotating magnetic flux compressors are tremendous in order to determine the time evolution of complex interacting electromagnetic fields, the transient mechanical stresses, and thermal regimes. However, the payoffs are equally tremendous. These machines can make the difference in assuring the success of high-power

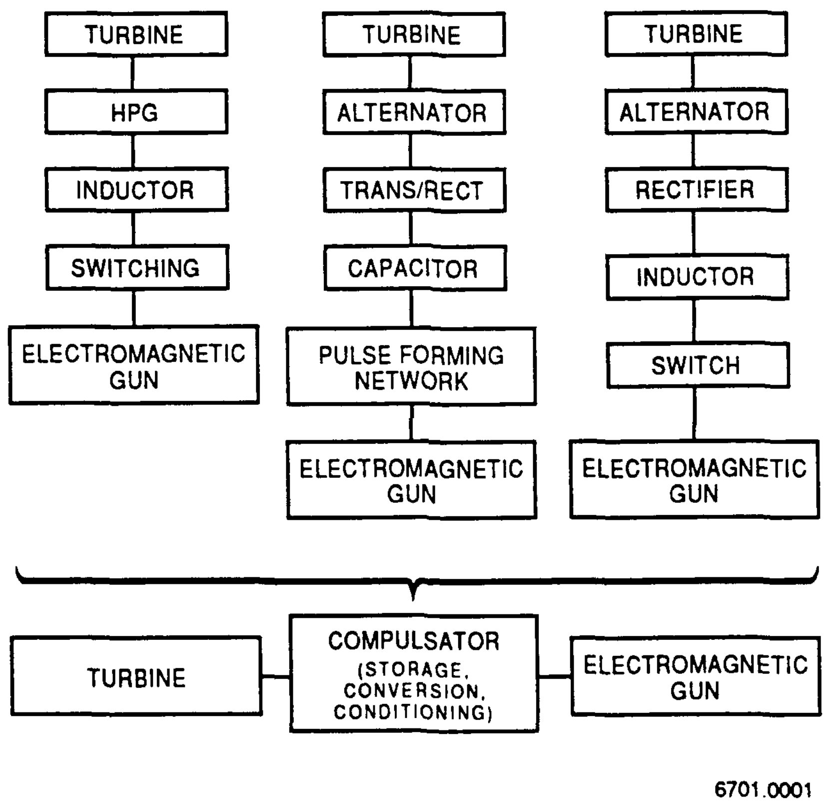


Figure 1. One-element power supply philosophy

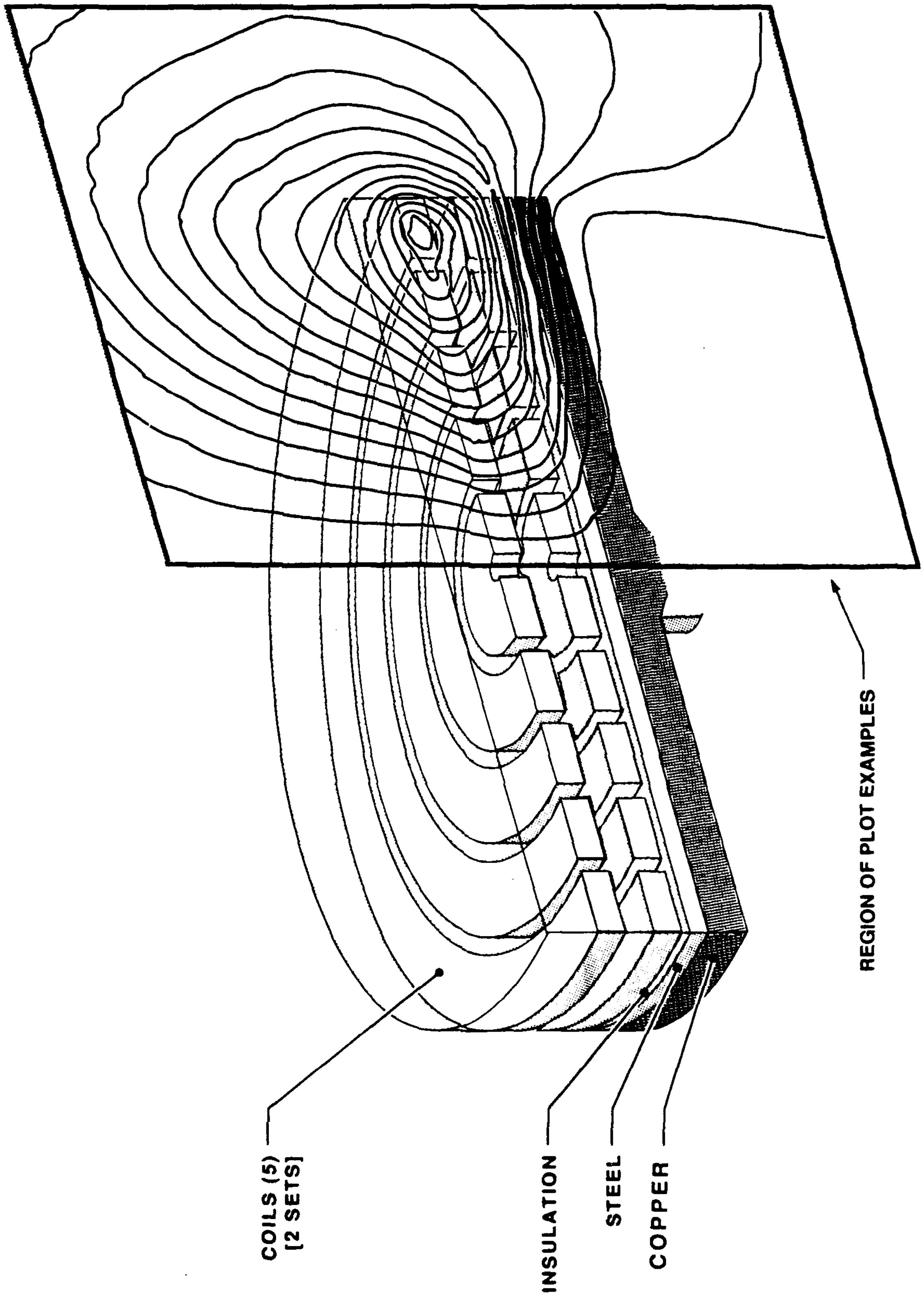
heteropolar accelerators for hypervelocities. On the other hand, since the date when the patent [14] for the compensated pulsed alternator, including the rotating flux compressor feature, was issued, advances in rotating flux compressors have been minor (Weldon, et al [37]).

A frequent misunderstanding in approaching pulsed rotating and linear electromechanical converters is to equate the compensation by eddy-currents produced in shields in order to reduce the internal impedance - during the pulsed discharge (by so-called "passive compensation") with the flux compression phenomena (incorrectly called "active compensation").

The first case is widely known and applied frequently in short-circuit alternators used for high-power circuit testing of electrical equipment [38]. A one-pulse compensated pulsed alternator, including a pulse-shaping system based on harmonic synthesis, was patented by Kapitza in 1927 and used by Rutherford and Kapitza to produce high magnetic fields [39, 40, 41]. The "Pulsator" machine [42] invented by Rebut and Torossian (1966) represents a more sophisticated "one-pulse" compensated alternator used for controlled thermonuclear fusion application. Enrico Levi and Harish Pande introduced a continuous aluminum shield for compensation and used an airgap-winding for their "electromechanical pulsers" at the Polytechnic Institute of Brooklyn (1961 and 1963) [43].

The principle of all these machines is simple and is illustrated (Figure 2) below for a compensated structure in which the two-layer winding is an airgap winding and the shield is continuous and made of aluminum. The

Figure 2. System topology



eddy-currents appearing in the shield, delay the penetration of magnetic flux, and consequently maintain the internal impedance of the system at a low level. Figures 3, 4, 5, and 6 show the unfolding in time of the compensation phenomenon at 5, 10, 15, and 20 µsec-time intervals.

The second case is the flux compression phenomenon and was proposed by the author as his part in a patent for pulsed alternators [14]. It is achieved in its simplest form by an identical winding connected in series with the armature winding and placed on the other member of the rotating machine.

This arrangement does not provide true compensation since, during a 360 electrical degree cycle, the machine takes all the possible values for the internal impedance from the maximum to the minimum. The misnomer "compensation" comes from the false analogy with a dc compensated machine, where a similar arrangement provides compensation due to the fact that the commutator and brush system maintains the alignment of magnetic axes of the two windings, in spite of the rotation of the armature. In the arrangement described below, the angle between the two magnetic axes is continuously varying, covering all the possible values for the internal inductance (considering that such a machine is "compensated" is analogous to stating that a broken watch shows the perfect time once every 12 hours).

The essential difference between "compensation" and "flux compression" is related to the electromechanical power conversion feature. In the second case, the system of two windings linking a magnetic flux in the conditions of

Figure 3. Magnetic flux at t = 5 μsec

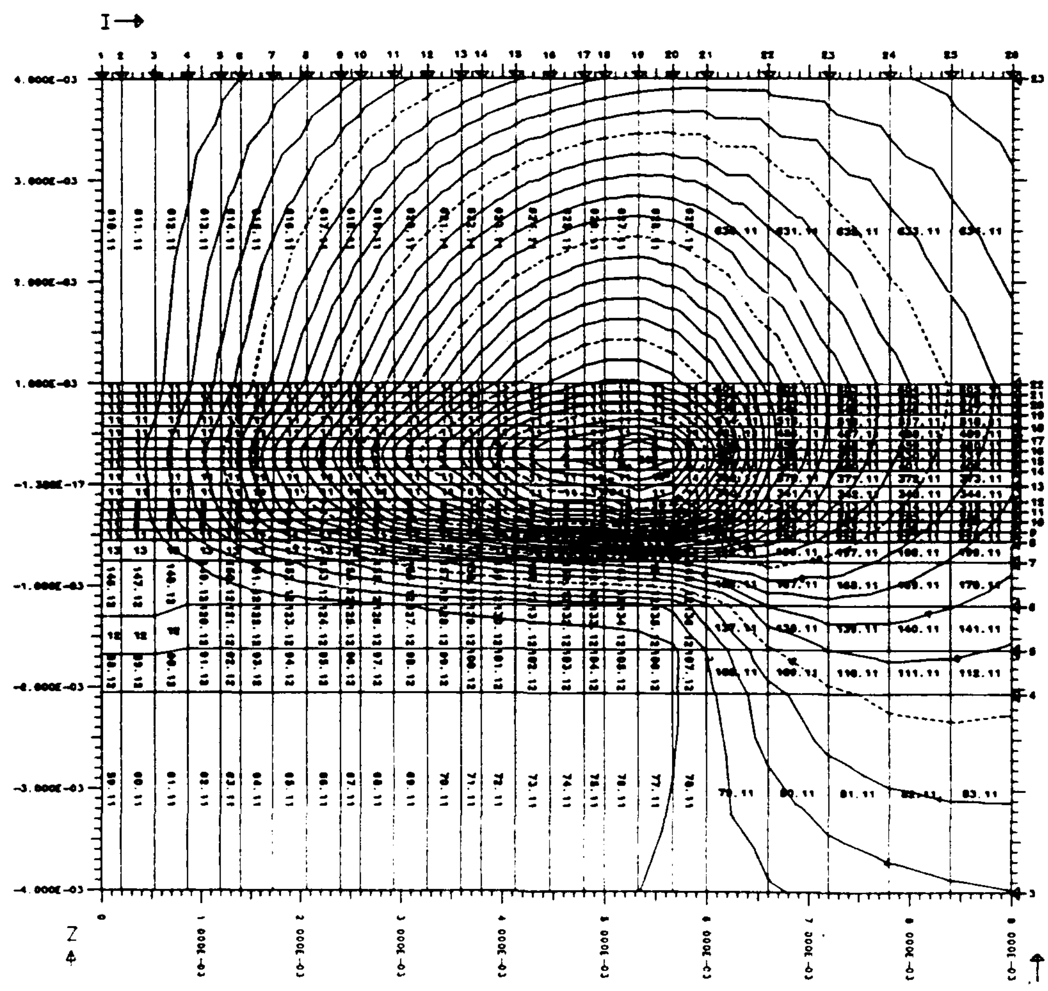


Figure 3a. Vector potential values at t = 5 μsec

Contour number	r X A	First point found r	z	in Elmement numbered
1	1.71027E-08	5.53221E-03	-4.00000E-03	77
2	3.42054E-08	6.60000E-03	-3.64420E-03	79
3	5.13080E-08	6.60000E-03	-2.38445E-03	79
4	6.84107E-08	7.20000E-03	-2.35157E-03	80
5	8.55134E-08	7.20000E-03	-1.78960E-03	109
6	1.02616E-07	7.20000E-03	-1.46341E-03	138
7	1.19719E-07	6.60000E-03	-1.07205E-03	166
8	1.36821E-07	7.20000E-03	-9.58662E-04	167
9	1.53924E-07	4.13333E-03	-5.81437E-04	188
10	1.71027E-07	4.46667E-03	-5.63228E-04	189
11	1.88129E-07	5.33333E-03	-5.68966E-04	192
12	2.05232E-07	4.13333E-03	-4.88142E-04	217
13	2.22335E-07	4.13333E-03	-4.62129E-04	217
14	2.39438E-07	4.46667E-03	-4.59439E-04	218
15	2.56540E-07	5.33333E-03	-4.71676E-04	221
16	2.73643E-07	5.66667E-03	-4.51507E-04	222
17	2.90746E-07	4.46667E-03	-3.78475E-04	247
18	3.07848E-07	4.46667E-03	-3.51094E-04	247
19	3.24951E-07	5.33333E-03	-3.74704E-04	250
20	3.42054E-07	4.46667E-03	-2.76764E-04	276
21	3.59156E-07	5.00000E-03	-2.52543E-04	278
22	3.76259E-07	5.33333E-03	-2.79454E-04	279
23	3.93362E-07	5.00000E-03	-1.53050E-04	307
24	4.10464E-07	5.33333E-03	-1.82612E-04	308
25	4.27567E-07	5.00000E-03	-2.19852E-05	336
26	4.44670E-07	5.33333E-03	-3.81961E-05	337
27	4.61772E-07	5.00000E-03	1.20807E-04	365
28	4.78875E-07	5.33333E-03	9.75367E-05	366
29	4.95978E-07	5.33333E-03	1.66569E-04	395
30	5.13080E-07	5.33333E-03	3.50000E-04	424

Figure 4. Magnetic flux at t = 10 μsec

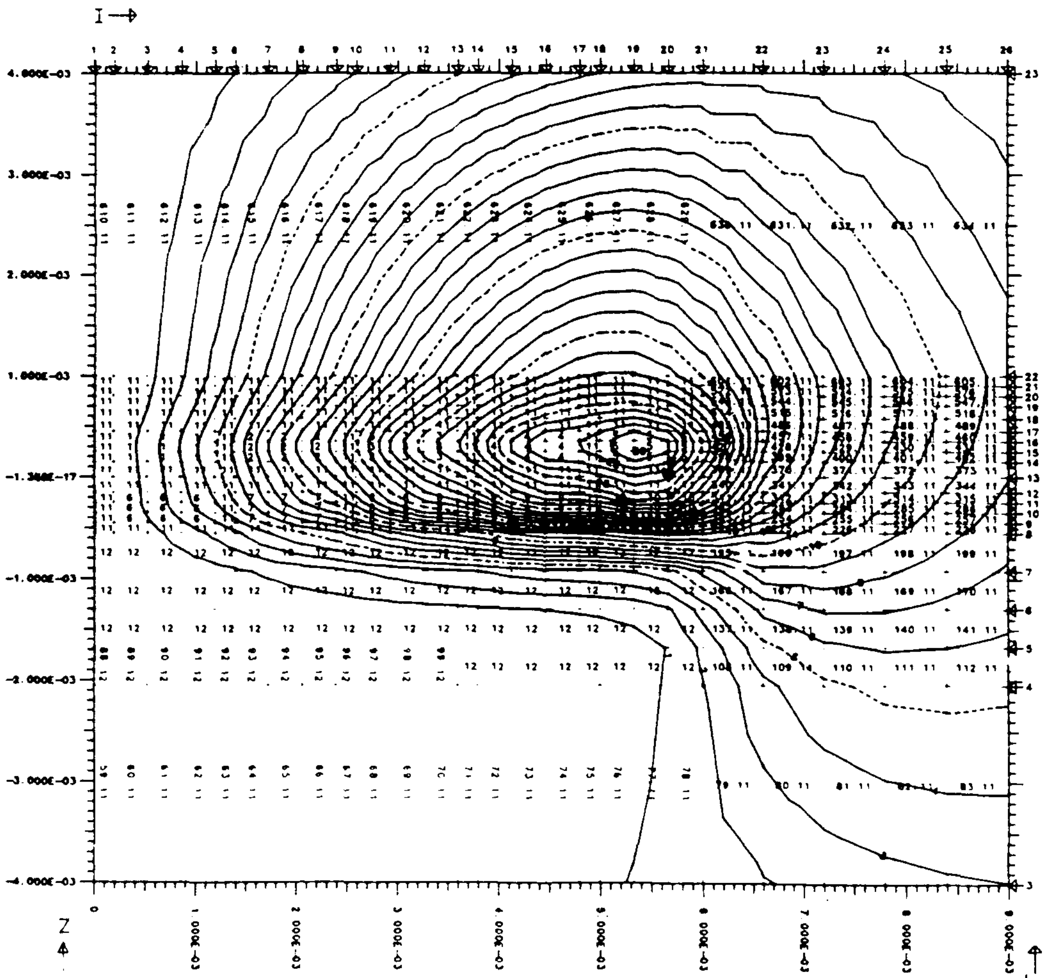


Figure 4a. Magnetic vector potential values at t = 10 μsec

Contour number	r X A	First point found r	z	In Elmement numbered
1	3.61410E-08	5.25737E-03	-4.00000E-03	76
2	7.22820E-08	6.19910E-03	-3.35292E-03	79
3	1.08423E-07	6.60000E-03	-2.84064E-03	79
4	1.44564E-07	7.20000E-03	-2.66588E-03	80
5	1.80705E-07	7.80000E-03	-2.22314E-03	81
6	2.16846E-07	7.80000E-03	-1.71033E-03	110
7	2.52987E-07	7.20000E-03	-1.32633E-03	138
8	2.89128E-07	6.60000E-03	-1.03483E-03	166
9	3.25269E-07	3.60000E-03	-5.69286E-04	186
10	3.61410E-07	4.13333E-03	-6.04573E-04	188
11	3.97551E-07	4.13333E-03	-5.56776E-04	188
12	4.33692E-07	4.80000E-03	-5.53115E-04	190
13	4.69833E-07	5.33333E-03	-5.59844E-04	192
14	5.05974E-07	4.13333E-03	-4.69568E-04	217
15	5.42115E-07	4.46667E-03	-4.68438E-04	218
16	5.78256E-07	5.00000E-03	-4.52977E-04	220
17	6.14397E-07	5.33333E-03	-4.60063E-04	221
18	6.50538E-07	4.46667E-03	-3.81746E-04	247
19	6.86679E-07	4.46667E-03	-3.52069E-04	247
20	7.22820E-07	5.33333E-03	-3.79866E-04	250
21	7.58961E-07	5.33333E-03	-3.52675E-04	250
22	7.95102E-07	5.33333E-03	-3.15237E-04	279
23	8.31242E-07	5.33333E-03	-2.76681E-04	279
24	8.67383E-07	5.33333E-03	-2.29912E-04	308
25	9.03524E-07	5.33333E-03	-1.64691E-04	308
26	9.39665E-07	5.33333E-03	-8.14006E-05	337
27	9.75806E-07	4.80000E-03	1.40253E-04	364
28	1.01195E-06	5.33333E-03	7.80595E-05	366
29	1.04809E-06	5.33333E-03	1.50401E-04	395
30	1.08423E-06	5.33333E-03	2.50000E-04	395

Figure 5. Magnetic flux at t = 15 μsec

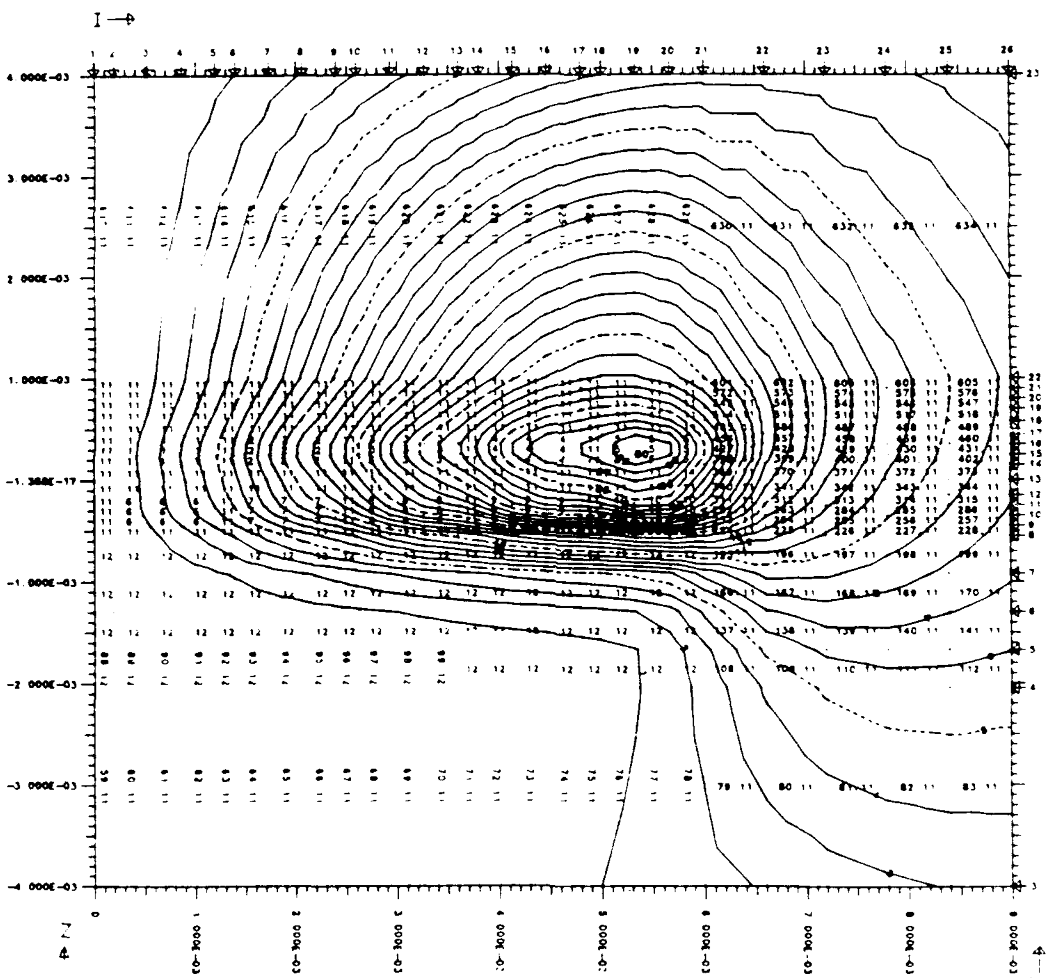


Figure 5a. Magnetic vector potential values at t = 15 μsec

Contour number	r X A	First point found r	z	in Elmement numbered
1	5.65845E-08	5.00353E-03	-4.00000E-03	76
2	1.13169E-07	6.00000E-03	-3.01951E-03	78
3	1.69753E-07	6.60000E-03	-3.14876E-03	79
4	2.26338E-07	6.60000E-03	-2.19407E-03	79
5	2.82922E-07	7.20000E-03	-2.16063E-03	80
6	3.39507E-07	7.20000E-03	-1.74285E-03	109
7	3.96091E-07	7.20000E-03	-1.44838E-03	138
8	4.52676E-07	6.60000E-03	-1.12952E-03	166
9	5.09260E-07	6.60000E-03	-9.75743E-04	166
10	5.65845E-07	3.60000E-03	-5.65848E-04	186
11	6.22429E-07	4.13333E-03	-6.07511E-04	188
12	6.79014E-07	4.13333E-03	-5.60372E-04	188
13	7.35598E-07	4.46667E-03	-5.52470E-04	189
14	7.92183E-07	5.33333E-03	-5.70756E-04	192
15	8.48767E-07	4.13333E-03	-4.66791E-04	217
16	9.05351E-07	4.46667E-03	-4.67511E-04	218
17	9.61936E-07	5.00000E-03	-4.53777E-04	220
18	1.01852E-06	5.33333E-03	-4.60876E-04	221
19	1.07510E-06	4.46667E-03	-3.74902E-04	247
20	1.13169E-06	5.00000E-03	-3.57630E-04	249
21	1.18827E-06	5.33333E-03	-3.75148E-04	250
22	1.24486E-06	4.46667E-03	-2.53143E-04	276
23	1.30144E-06	5.33333E-03	-3.02839E-04	279
24	1.35803E-06	5.33333E-03	-2.61292E-04	279
25	1.41461E-06	5.33333E-03	-1.97859E-04	308
26	1.47120E-06	5.00000E-03	-9.46264E-06	336
27	1.52778E-06	5.33333E-03	-1.89386E-05	337
28	1.58437E-06	5.33333E-03	6.37434E-05	366
29	1.64095E-06	5.33333E-03	1.42674E-04	366
30	1.69753E-06	5.33333E-03	2.50000E-04	395

Figure 6. Magnetic flux at t -= 20 μsec

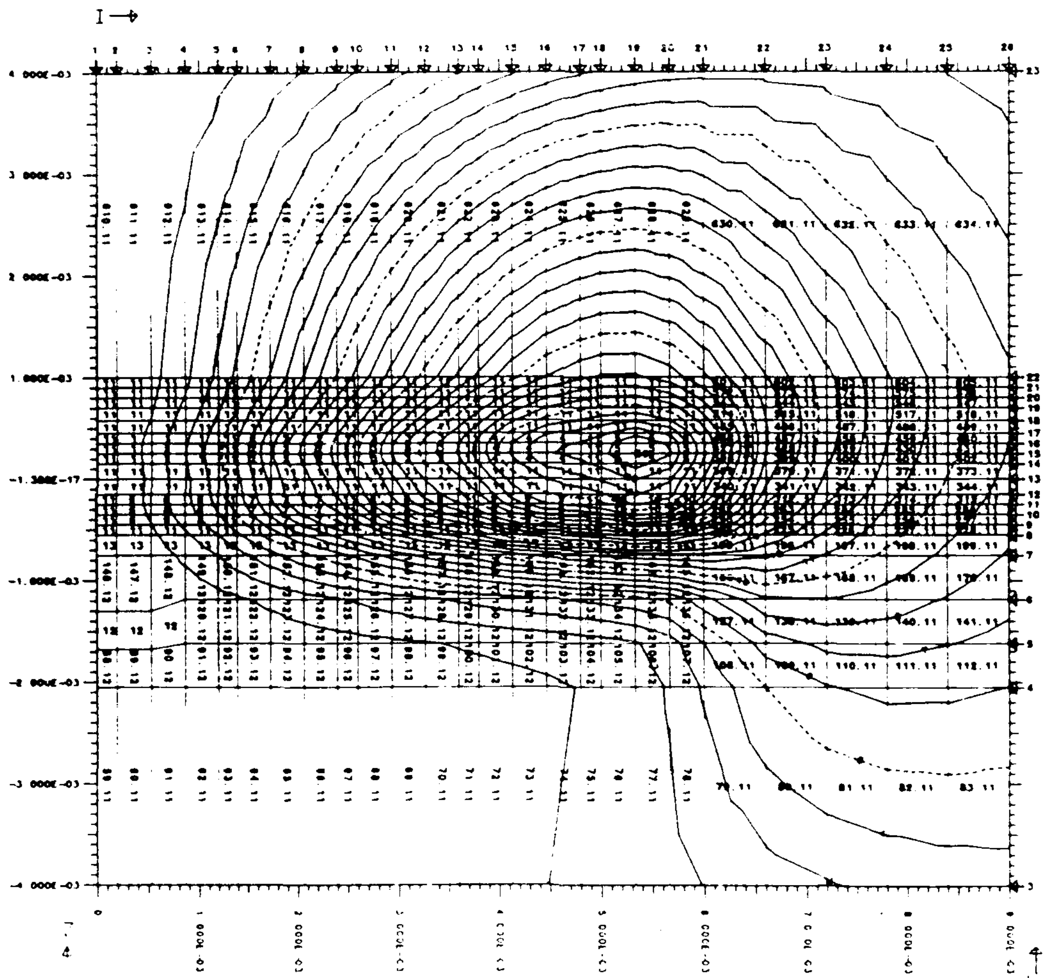


Figure 6a. Magnetic vector potential value at t = 20 μsec

Contour number	r X A	First point found r	z	in Elmement numbered
1	8.72344E-08	4.49038E-03	-4.00000E-03	74
2	1.74469E-07	5.66667E-03	-2.61419E-03	77
3	2.61703E-07	6.00000E-03	-2.27288E-03	78
4	3.48937E-07	6.60000E-03	-2.83417E-03	79
5	4.36172E-07	6.60000E-03	-2.05208E-03	79
6	5.23406E-07	7.80000E-03	-2.20496E-03	81
7	6.10640E-07	7.20000E-03	-1.70583E-03	109
8	6.97875E-07	6.60000E-03	-1.34188E-03	137
9	7.85109E-07	3.60000E-03	-7.62788E-04	157
10	8.72344E-07	4.13333E-03	-7.90858E-04	159
11	9.59578E-07	4.46667E-03	-7.71904E-04	160
12	1.04681E-06	5.00000E-03	-7.55330E-04	162
13	1.13405E-06	3.60000E-03	-5.72009E-04	186
14	1.22128E-06	3.80000E-03	-5.61852E-04	187
15	1.30852E-06	4.13333E-03	-5.82448E-04	188
16	1.39575E-06	4.46667E-03	-5.82540E-04	189
17	1.48298E-06	4.80000E-03	-5.62034E-04	190
18	1.57022E-06	5.33333E-03	-5.66542E-04	192
19	1.65745E-06	4.46667E-03	-4.76029E-04	218
20	1.74469E-06	4.80000E-03	-4.51181E-04	219
21	1.83192E-06	5.33333E-03	-4.68557E-04	221
22	1.91916E-06	4.46667E-03	-3.59542E-04	247
23	2.00639E-06	5.33333E-03	-3.96905E-04	250
24	2.09362E-06	5.33333E-03	-3.59731E-04	250
25	2.18086E-06	5.33333E-03	-3.09463E-04	279
26	2.26809E-06	5.33333E-03	-2.54552E-04	279
27	2.35533E-06	5.33333E-03	-1.55267E-04	308
28	2.44256E-06	5.33333E-03	-6.47236E-06	337
29	2.52980E-06	5.33333E-03	1.06247E-04	366
30	2.61703E-06	5.33333E-03	2.50000E-04	395

a large inductance and consequently small current is compressed by external forces, converting mechanical (kinetic) power into electrical power - in condition of limited flux losses - until it reaches the position of minimum inductance in the presence of large currents.

2.2 SIMPLIFIED FLUX COMPRESSION CIRCUIT ANALYSIS AND ELECTROMECHANICAL POWER CONVERSION CONSIDERATIONS

The classical treatment of the simplest flux compression system involves a simple R, L circuit, in which the flux to be compressed is /stored initially in the inductance by means of an external power supply which in Figure 7 is represented as a current source which is already short-circuited at t=0 when the compression begins. In principle, the flux is compressed by rapidly modifying the relative position of an arrangement of conductors [15], forcing the magnetic flux to conform to a continuously decreasing inductance $L(t)$. Since the magnetic energy (as well as the magnetic coenergy) in the system, W_m, can be expressed as a function of the instantaneous current, I(A), and the magnetic flux, $\Psi(Wb)$: as state variables:

$$W_m = \frac{1}{2}L(t)I^2 = \frac{1}{2}\frac{\Psi^2}{L(t)} \tag{2.1}$$

it can be seen that in condition of constant flux, the magnetic energy stored in the system and partly delivered to the load is continuously increasing until

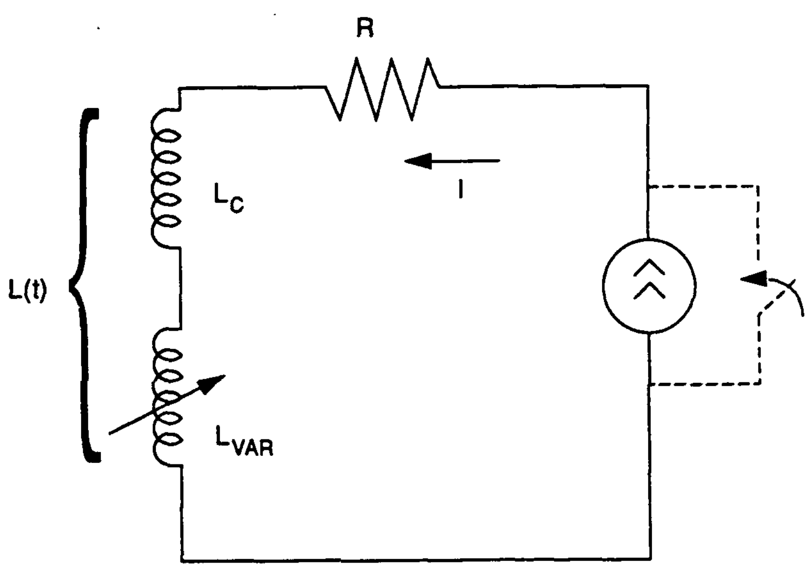


Figure 7. The simplest scheme for a flux compressor

the minimum inductance position is reached. It is obvious that the increase in electromagnetic energy occurs by conversion of the mechanical energy stored as kinetic energy in the moving conductors. However, the flux does not remain constant, being affected by losses into the circuit resistance, R.

The differential equation for the circuit in terms of magnetic flux is:

$$\frac{d(LI)}{dt}+\frac{R}{L}(LI)=0, \tag{2.2}$$

where the magnetic flux Ψ has been expressed in terms of inductance:

$$\psi = LI(Wb) \tag{2.3}$$

The solution of the differential equation (2) is:

$$LI = L_O I_O \exp\left\{\int_o^t\left(\frac{R_L}{L}\right)dt\right\} \tag{2.4}$$

where

L = total inductance of the circuit at time t, (H)

I = current through the circuit at time t, (A)

L_0 = initial total inductance at time $t = 0$, (H)

I_0 = initial current (time t=0), (A)

R_L = the load resistance which somewhat increased to take into account the resistance of the rest of the circuit [Ω.]

As the total inductance is decreasing, the current is increasing:

$$I(t) = \frac{L_O}{L(t)} I_O \exp\left\{-\int_o^t \left(\frac{R_L}{L(t)}\right) dt\right\} \tag{2.5}$$

In order to obtain the instantaneous power balance, we multiply (2.2) by I, and by rewriting the first term, we get:

$$-\frac{1}{2} I^2 \frac{dL}{dx} \bullet \frac{dx}{dt} = RI^2 + \frac{d}{dt}\left(\frac{1}{2} LI^2\right) \tag{2.6}$$

Considering that the flux compression is obtained by a translational motion in the direction of the generalized coordinate, x,, we obtain:

$$-\left[\frac{1}{2} I^2 \frac{dL}{dx}\right]\left[\frac{dx}{dt}\right] = RI^2 + \frac{d}{dt}\left(\frac{1}{2} LI^2\right), \tag{2.7}$$

where the two pairs of brackets in the left hand member contain the generalized force on the x direction $\overline{F}$ and the velocity in the same direction, $\overline{u}$,respectively:

$$-\bar{F} \bullet \bar{u} = RI^2 + \frac{dW}{dt} \tag{2.7'}$$

The mechanical power input necessary to decrease the inductance of the circuit is spent in increasing the magnetic energy stored in the system and is partially lost in Joule heat.

If, as in rotating machinery, a rotational motion is responsible for the flux compression by decreasing the inductance of the circuit, relation (2.7) becomes:

$$-\left[\frac{1}{2}I^2\frac{dL}{d\alpha}\right] \bullet \left[\frac{d\alpha}{dt}\right] = RI^2 + \frac{d}{dt}\left(\frac{1}{2}LI^2\right) \tag{2.8}$$

where, in the left side we have the torque $\bar{T}(N \bullet m)$ as generalized force in the α direction and the angular velocity $\bar{\omega}$ rad/s) as the derivative of the angular displacement αrad with respect to time. Then the instantaneous power balance becomes:

$$-\bar{T} \bullet \bar{\omega} = RI^2 + \frac{dW_m}{dt} \tag{2.9}$$

where W_m is the magnetic energy stored in the circuit at that particular moment.

However, pulsed electrical generators incorporating flux compression features have the advantage of producing new flux during compression which is continuously integrated and compressed, in addition to the initial flux. Then the equation (2.2) becomes

$$\frac{d}{dt}[LI] + \frac{R}{L}[LI] = f(t) \qquad (2.2')$$

The simplicity of the equation is deceiving, $f(t)$ being a complicated function, since the excitation flux and the angular velocity, ω, are variable and nonlinear during the compression and the differential equation must be linked to the instantaneous balance of power, and to the diffusion equation for the electromagnetic fields in 3D structures requiring elaborate numerical codes, in condition of relative motion.

In equations (2.7-2.9) the converted power appearing in the left side member has a negative sign. In electromagnetic converters, this sign shows the direction of energy flow. This case indicates the generator - flux compressor regime in the linear topology variant, equations (2.7) and (2.7') and in the rotating variant, equations (2.8) and (2.9).

The mechanical energy comes from the kinetic energy usually stored in the mass of moving conductors achieving compression. In turn, the kinetic energy is produced by some form of primary energy coming from an explosive for the common flux compressors designed to achieve ultra-high fields or from a turbine, in the case of electromechanical power supplies for launchers (e.g. compulsator) [14].

Conversely, for electromagnetic accelerators, the terms describing the converted power and the magnetic energy stored in the circuit change signs in order to show the change in direction of energy flow, from magnetic energy to an increase in the kinetic energy of the projectile:

$$\overline{F} \bullet \overline{u} = RI^2 - \frac{dW_m}{dt} \tag{2.7''}$$

and

$$\overline{T} \bullet \overline{\omega} = RI^2 - \frac{dW_m}{dt} \tag{2.9'}$$

The equations (2.1) to (2.9) are global or integral relations describing the whole magnetic flux compression (generators) or accelerator circuits. Point (or local) relations may be used for a local and distributed characterization of the power conversion.

Following the classical treatment of Levi and Panzer [16] Nasar [17], or White and Woodson [18], the unit volume element of the conductor is considered the building block of the converter. By establishing the different specific power densities and by multiplying them by the elementary volume *dw*, the electromechanical energy conversion process is characterized and defined locally, in each point of the converter.

The power balance relations for the unit volume of conductor [16] are:

$$\zeta \ \overline{u} \bullet \frac{d\overline{u}}{dt} = (\overline{J} \times \overline{B}) \bullet \overline{u} + \bar{f}_m \bullet \overline{u} \tag{2.10}$$

$$\frac{\overline{J}^2}{\sigma} = \overline{E} \bullet \overline{J} + (\overline{u} \times \overline{B}) \bullet \overline{J} \tag{2.11}$$

We denote the power per unit volume or specific power with lower case $p(w/m^3)$. Then, p_{kin} = specific kinetic power, p_{conv}= specific converted

power transformed into heat by Joule effect. Then:

$$p_{kin} = f\,\bar{u}\frac{d\bar{u}}{dt} = \frac{d}{dt}\left(\frac{1}{2}\zeta\, u^2\right) \tag{2.12}$$

$$p_{conv} = (\bar{J} \times \bar{B})\bar{u} \tag{2.13}$$

$$p_{mech} = \bar{f}_m \bullet \bar{u} \tag{2.14}$$

$$p_{el} = \bar{E} \bullet \bar{J} \tag{2.15}$$

$$p_{loss} = \frac{J^2}{\sigma} \tag{2.16}$$

leading to the equivalent circuits per unit volume of conductor of a magnetic flux compressor (generator) (Figure 8) and of an electromagnetic accelerator (Figure 9).

What must be the law of variation for velocity $\bar{u}$? One of the optimum situations in electromagnetic accelerator design is reached when the forces are uniformly distributed over the projectile - assuring a uniform distribution of stresses - and leading to a constant acceleration during the launch time. In [8] (Driga, et.al.), the means to achieve an accelerated travelling wave following almost exactly the motion law of the projectile by using a continuously rising frequency or a variable pitch winding or a combination of both was described.

As shown in Driga [9], a rising frequency generator, while possible in different variants in a pulsed mode, is a very demanding and complex electrical machine which works properly only for a limited span of its

circumference. But even then, such a power supply would not create the purely sinusoidal travelling wave acting uniformly on the currents in the projectile - ideal condition which is unattainable in practice. The reasons for that [19] are related to the harmonics arising due to the finite length of the barrel wave packet, its lack of perfect smoothness, and the finite length of the projectile whose leading and lagging ends must slip ahead and behind the wave packet, only the middle of it travelling at the speed of the accelerated wave.

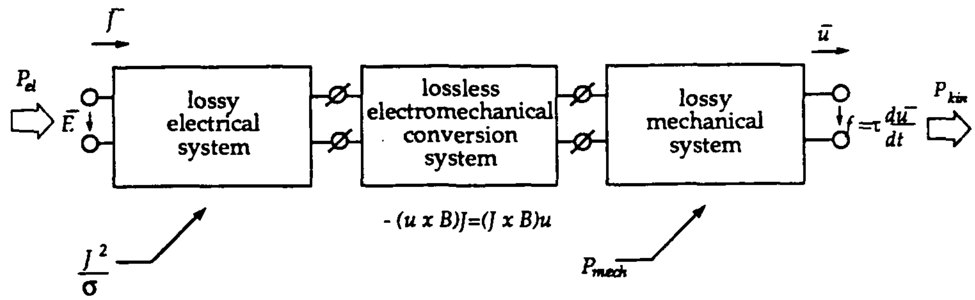


Figure 8 Electromechanical power conversion in a unit volume of an electromagnetic flux compressor

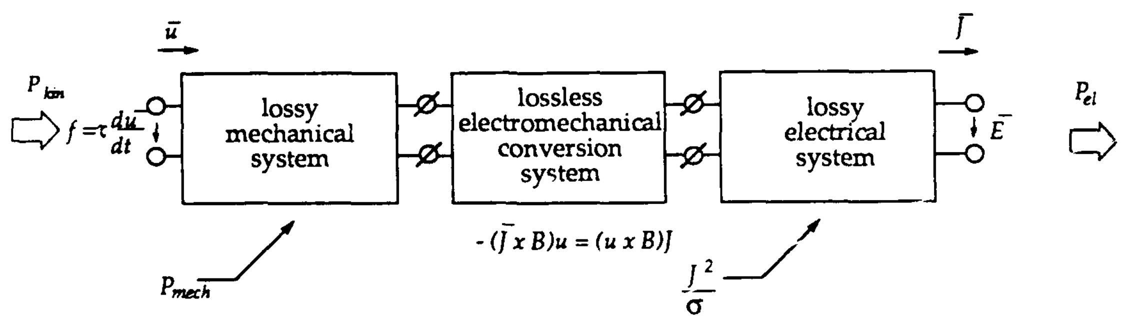


Figure 9. Electromechanical power conversion in a unit volume of an electromagnetic launcher

2.3 ANALYTICAL AND NUMERICAL TOOLS FOR ANALYSIS AND SYNTHESIS OF ELECTROMAGNETIC LAUNCHERS AND POWER SUPPILES

Then, what are the right tools for the analysis and synthesis of power supplies and electromagnetic accelerators? Obviously, the rapid evolution of the magnetic flux distributions and currents accompanying them needs a global characterization; while locally, the evaluation of the flux densities and current densities describe the electromagnetic state at each point.

It is widely assumed that the next breakthroughs and innovations in hypervelocity accelerator technologies will come from advances in analysis and modelling, controlling electromagnetic diffusion and fast transient phenomena in the projectile and launcher barrel, and in the power supplies. This leads us to consider the finite element method as the main tool for analyzing the transient electromagnetic phenomena during the magnetic flux compression - expansion.

Later in Chapter 3 the above problem, involving a 3-D f.e.m. formulation using a Galerkin technique, is described in detail.

Driga, et. al., [10] and Pillsbury [24] describe a 2-D axisymmetric f.e.m. formulation of a transient electromagnetic diffusion in fast discharging homopolar machines, also used to illustrate simple magnetic compensation arrangements.

Magnetic flux compression - expansion devices require the knowledge of 3-D flux distributions, and since the magnetic flux and, consequently, magnetic stress concentrations occur at connections where the field is three-dimensional, simplifications to two-dimensional structures are no longer acceptable.

The computational efforts for a three-dimensional f.e.m. electromagnetic computer code are very large since such a transient code solves the problems for all three components of the vector potential (A_x, A_y, A_z).

By comparison, the similar code for two-dimensional axisymmetric structures mentioned above solves the problem for only one component of the magnetic vector potential (A_Φ).

One of the most important features, important not only for numerical reasons, but having also deep theoretical implications, is the selection of the Coulomb's gauge for the magnetic potential:

$$div\overline{A} = 0 \tag{2.12}$$

This choice, beside completely defining the field of vectors $\overline{A}$, makes the statement that the quasistationary assumption $\frac{\partial \overline{D}}{\partial t} = 0$ holds. Also, that locally:

$$div\overline{J} = 0 \tag{2.13}$$

is valid for all the magnetic flux compression - expansion devices (or, globally, in circuit terms, the first Kirchhoff's law applies). Equation (2.12) is immediately derived from (2.13), taking into account that the product $R^2|\overline{A}|$ remains finite at large distances ($R \rightarrow \infty$). Coulomb's gauge choice, required point-wise, is justified by Driga, et. al. [10] and Pillsbury [24] as simplifying the governing equations for the 2-D formulation, and its application. In that particular code, was imposed by the use of Lagrange multipliers. However, in the axisymmetric geometry, the condition follows naturally, since prescribing a current, the entire circuit is implied, and condition (2.13) is respected. Also, in such conditions, the Lagrange multipliers may be replaced equivalently by some appropriate penalty function.

In the 3-D finite element code, the situation changes completely. In such geometries, the usual forcing function is prescribed brick by brick, and small discrepancies introduced by round-off errors, for example, lead to the fact that Coulomb's gauge becomes invalid. Also, the nonidentical three-dimensional elements joining each other add to discrepancies in the condition (2.12), point-wise.

The tool imposing the Coulomb's gauge constraint globally per element and, also, point-wise in the sense of distribution along two adjacent integration points, is the use of Lagrange multipliers.

As shown in Chapter 4, the Lagrange multipliers in this three-dimensional code represent sheets of currents flowing through the faces of the three-dimensional elements and, by compensating the discrepancies,

impose the Co'ılomb's gauge, point-wise, in the entire volume. The numerical and theoretical implications of this fact represent some of the original contributions of this work.

The nonuniformities in prescribing the forcing function and other nonuniformities introduced by the nature of the finite element method itself (finite elements for example...) are compensated by the magnetic flux produced by surface currents flowing through the faces of the volume elements. Then, in Chapter 5, this compensation method is applied to the 3-D finite element formulation for solving the complete system of Maxwell's equations, under the constraint $divA = 0$. The three-dimensional, transient code is applied to a generalized Bondoletov-type accelerator, having both massive conductive elements characterized by free diffusion and stranded and transposed conductive elements, in which the flow of current is controlled. The magnetic field and current density space distributions are plotted at successive intervals for a rectangular stator coil repelling a rectangular copper plate.

Comparisons are made with a similar flux compression system with round coil and round plate which, due to its axisymmetric arrangement, can be solved with a two-dimensional code. The difference between the two solutions is theoretically fundamental, being based on the manner in which the constraint $divA = 0$ is imposed.

In the latter case, it follows almost automatically due to the symmetry of the current (and vector potential) distributions; in the first case, it practically illustrates the considerations made in Chapter 4, showing how

efficient are the limit cases of sheets of currents flowing along the faces of the volume-finite elements in establishing the right (quasistationary) distribution of magnetic flux.

The computational effort for analyzing and synthesizing magnetic flux compression - expansion convertors using using three-dimensional f.e.m. transient codes is tremendous. The payoffs are equally tremendous, since only perfectly designed defices can assure the success of hypervelocity accelerators, overcoming the extreme stress levels with an exact knowledge of their transient evolution.

The examples presented are three-dimensional plots of magnitude and directions for magnetic field and currents density of a flux compression system with transient field diffusion. The consecutive stages of the magnetic flux compression are shown at consecutive intervals of 5 μsec, namely 5, 10, 15, and 20 μsec. (During this time the inertia causes the projectile to move very little.)

There is another important reason why the two-dimensional axisymmetric codes are ineffective and must be replaced by three-dimensional ones (even if apparently small changes in structure can be made to the latter to look axisymmetric), namely the armature reaction. The armature reaction, changing in time during launching due to its spatial shift of the magnetic fields, adds irrevocably the third dimension component.

The output of the 3-D code, in terms of magnetic vector potential, permits the calculation of magnetic fields and current density distributions,

electromagnetic body forces, Joule losses, and the density of magnetic energy stored in the field. Then, all terms in the formulas (2.12) to (2.16) can be calculated and the electromechanical power conversion characterized in each point of the electromagnetic device.

Globally, by integration, the magnetic flux being compressed results almost directly, since the vector potential, $\overline{A}$, and the flux density, $\overline{B}$, are related by the Stokes' theorem:

$$\Phi = \iint_{S_c} \overline{B} \bullet \overline{ds} = \oint_c \overline{A} \bullet d\ell$$

where the surface S_c lies on the curve c.

Similarly, the currents, the total magnetic energy stored in the field, total Joule losses, and global forces can be calculated from local field values and integrated.

Then, the formulas (2.5) to (2.9) may be applied characterizing the power conversion phenomena globally in the magnetic flux compression - expansion devices.

In fact, the forces of electromagnetic origin (and the velocities which result from the continuous acceleration in the barrel) are of main concern in obtaining new limits of performance in electromagnetic hypervelocity accelerators. Such forces can be calculated in three different ways.

1. As the global, resultant force (a generalized Lagrangian force, acting along a generalized coordinate);

2. As the distribution of body forces in the projectile volume and in the rest of the accelerator;

3. And, finally, as stresses, using the artificial method of electromagnetic stress tensor, reducing the problem of electric and magnetic fields of force to that of an elastic continuum.

The three methods are formally equivalent and their expressions result directly and elegantly due to the almost symmetrical structure of Maxwell's equations for macroscopic nonrelativistic electrodynamics.

The theoretical tools which transform the expressions for forces from one form to another are related to:

a) The constancy of magnetic flux as the constraint used to assure the proper transformation in order to find the generalized Lagrangian forces along a generalized coordinate (global or integral characterization); and

b) The complex and subtle notion of flux derivative (2.12) as applied to electromagnetic systems in motion. Again, using the constancy of the magnetic flux as a constraint, we obtain all the equivalent body forces of electromagnetic origin (local or point characterization).

Such results obtained in Chapter 5 permit a direct transition from the global to local (and reverse from local to global) characterization of forces in electromagnetic accelerators and their power supplies.

CHAPTER 3

THE FORMULATION OF THE FINITE ELEMENT, THREE DIMENSIONAL TRANSIENT ELECTROMAGNETIC PROBLEM FOR MAGNETIC FLUX COMPRESSION DEVICES AND ELECTROMAGNETIC LAUNCHERS

3.1 INTRODUCTION

The finite element method is ideal for analyzing the transient magnetic field diffusion in the flux compression devices in three-dimensional domains. It can easily handle discontinuous geometrical shapes and material discontinuities in solving the penetration of magnetic flux into complex structures of the flux compression devices.

A formulation of three-dimensional transient electromagnetic problem for flux compression is presented in this chapter. It solves the magnetic field diffusion problems in terms of all three components of the magnetic vector potential (A_x, A_y, A_z) by comparison with the similar numerical codes for two-dimensional axisymmetric electromagnetic structures which solve the flux penetration problems for one component of the magnetic vector potential (A_Φ).

Magnetic flux compression devices comprise, among various conductive and insulating materials, also stranded and transposed conductors. These components are also taken into account by the present formulation by the artifice of considering their conductivity zero and prescribing their current densities as a function of time, usually obtained

with good accuracy using circuit analysis methods, or, alternatively, from an iteration procedure.

One of the important characteristics of the present formulation is the introduction of superficial currents - flowing through the faces of the three-dimensional finite elements - as Lagrange multipliers.

The approximation of the magnetic flux which is a perfect continuum due to the solenoidal characteristics of the flux density *B*, by the f.e.m, a discrete method of solving parabolic, partial differential equations [30], is performed by imposing the Coulomb gauge [31], pointwise. This is based on the fact (shown in Chapter 4) that the magnetic field produced by a distribution of volume currents *inside an element* (brick, prism, etc.) can be replaced by an identical magnetic field structure produced by a distribution of surface currents [26], [31] flowing on different faces of the respective elements. The prescription of current densities element by element, round-off errors in calculating the current densities, the slight mismatch of adjacent or boundary elements, etc., will require a strong imposition of Coulomb's gauge in order to enforce the solenoidal character of the current density everywhere.

3.2 DOMAIN OF DEFINITION AND GENERAL ASSUMPTIONS

The domain under consideration, extending eventually to infinity in all three dimensions, has (Figure 10) three different types of subdomains:

D_J = conductors, carrying currents impressed by external or internal voltage or current sources (conductors stranded and transposed);
D_E = solid conductors which are seats of eddy currents;
D_0 = nonconductive domains.

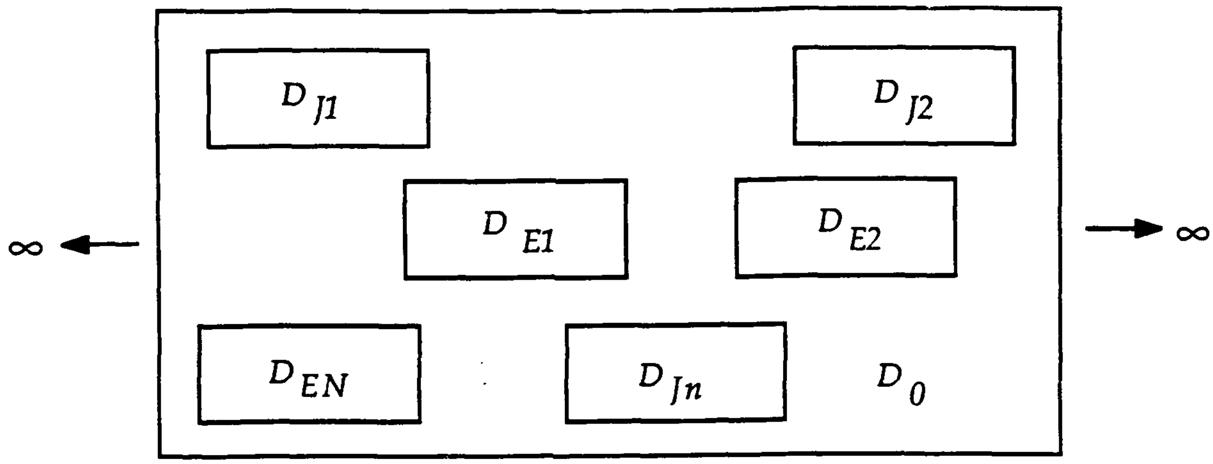


Figure 10. Structure of the electromagnetic system analyzed by the 3-D, transient f.e.m. code

The general assumptions are:

a. The magnetoquasistatic approximation applies to all electromagnetic launcher problems:

$$\frac{\partial \overline{D}}{\partial t} = 0 \tag{3.1}$$

b. The materials are isotropic.

c. Hysteresis effects are neglected.

3.3 MAXWELL'S EQUATIONS

Maxwell's equations, as applied to electromagnetic launchers, are:

$$curl\overline{H} = \overline{J} \tag{3.2}$$

$$curl\overline{E} = -\frac{\partial \overline{B}}{\partial t} \tag{3.3}$$

$$div\overline{B} = 0 \tag{3.4}$$

$$div\overline{J} = 0 \tag{3.5}$$

$$\overline{B} = \mu\overline{H} \tag{3.6}$$

$$\overline{J} = \sigma\left(\overline{E} + \overline{E}^1_{imp} + \overline{E}^2_{imp} \ldots\right) = \overline{J}_e + \overline{J}^1_s + \overline{J}^2_s + \ldots \tag{3.7}$$

where the "impressed" fields:

$$\overline{E}^i_{imp}(i = 1,2,\ldots n) \tag{3.8}$$

correspond to "impressed" electromotive forces of nonelectric origin (due to chemical effects, thermal effects, etc.).

In the case of electromagnetic launchers, persistent currents induced in the projectile in the earlier stages of launching, as well as the electromotive forces of electromechanical origin obtained by integrating the $\left(\overline{u} \times \overline{B}\right)$ induction terms, must be taken into account properly as being "impressed."

3.4 GOVERNING EQUATIONS

A theorem due to Helmholtz [25], [32] states that any vector field which is finite, uniform, and continuous and which vanishes at infinity may be expressed as the sum of a gradient of a scalar Ψ and a curl of a zero divergence vector $\overline{A}$:

$$\overline{F} = grad\psi + curl\overline{A}$$

under the condition:

$$div\overline{A} = 0 \tag{3.9}$$

then, this separation into a divergenceless part, curl $\overline{A}$, and a curless part, grad ψ, is unique [25], [32]:

$$\overline{B} = curl\overline{A} \tag{3.10}$$

The Coulomb's gauge applies in order that the continuity equation be satisfied in the domain of interest. It is enforced in the f.e.m. formulation by use of Lagrange multipliers, as shown in the next chapter.

Faraday's law is expressed (3.3) as:

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

However, when, in a body (conductor or not), the domain in which the field $\overline{E}$ is calculated moves with velocity $\overline{u}$ (as in the case of the projectile of the electromagnetic launcher), then the derivative of the magnetic flux must be considered in a context of an integration surface moving with the material [25].

Applying the general formula of the flux derivative [25] to the Faraday's law (3.3), we obtain:

$$\frac{d}{dt}\iint_s \overline{B} \bullet d\overline{s} = \iint_s \left[\frac{\partial \overline{B}}{\partial t} + \overline{u}\,div\overline{B} - curl\left(\overline{u} \times \overline{B}\right)\right] \bullet d\overline{s} \tag{3.11}$$

and, since $div\overline{B} = 0$, we obtain:

$$curl\bar{E} = -\frac{\partial \bar{B}}{\partial t} + curl\left(\bar{u} \times \bar{B}\right) \tag{3.11a}$$

and, using (1.9)

$$curl\bar{E} = -\frac{\partial}{\partial t} curl\bar{A} + curl\left(\bar{u} \times \bar{B}\right) \tag{3.11b}$$

Since the inversion of the order of the space and time derivatives is permissible,

$$curl\bar{E} = -curl\left[\dot{\bar{A}} + \bar{u} \times \bar{B}\right] \tag{3.11c}$$

or

$$curl\left(\bar{E} + \dot{\bar{A}} + \bar{u} \times \bar{B}\right) = 0 \tag{3.11d}$$

meaning that the expression in the parentheses is curlless and, consequently, may be expressed as a gradient of a scalar potential

$$\bar{E} + \dot{\bar{A}} + \bar{u} \times \bar{B} = -grad\psi \tag{3.12}$$

We may consider a "generalized" time derivative [25] of the vector potential:

$$\underset{-}{\dot{\bar{A}}} = \dot{\bar{A}} + \bar{u} \times \bar{B} \tag{3.12a}$$

such that

$$\bar{E} + \underset{-}{\dot{\bar{A}}} = -grad\psi \tag{3.12b}$$

In fact, $grad\psi$ may contain different terms

$$grad\psi = grad\psi_1 + grad\psi_2 + ... \tag{3.12c}$$

Then, the current density is:

$$\bar{J} = \sigma(\bar{E} + \bar{E}_{imp}) = \sigma\left(-\dot{\underline{\bar{A}}} - grad\psi\right) + \bar{J}_s = \bar{J}_e + \bar{J}_s \tag{3.13}$$

By using (3.13), the magnetic circuit law (3.2) may be written as:

$$curl\bar{H} = \bar{J} = \sigma\left(-\dot{\underline{\bar{A}}} - grad\psi\right) + \bar{J}_s, \tag{3.13a}$$

and by rearranging the terms, we obtain:

$$\sigma\dot{\underline{\bar{A}}} + \sigma grad\psi + curl\frac{1}{\mu}curl\bar{A} = \bar{J}_s \tag{1.13b}$$

In the stator, and in the projectile, as well as a corollary of Coulomb's gauge, the current density is solenoidal, (3.5).

Thus, by considering equation (3.13), we can write:

$$div\left[\sigma\left(-\frac{d\bar{A}}{dt} - grad\psi\right) + \bar{J}_s\right] = 0 \tag{3.14}$$

Using, from now on as shorthand, the del (∇) operator and considering, also, that $\bar{A}$ implies $\underline{\bar{A}}$, we can write:

$$\nabla \bullet \left(-\sigma\frac{d\bar{A}}{dt}\right) + \nabla \bullet (-\sigma\nabla\psi) - \nabla \bullet \bar{J}_s = 0 \tag{3.14b}$$

Summarizing the mathematical transformations performed above, the finite element, three dimensional formulation of the transient

electromagnetic problem is governed by the following system *(I)*of differential equations with partial derivatives:

$$(I)\begin{cases} \sigma\dfrac{d\overline{A}}{dt}+\nabla\times\dfrac{1}{\mu}\nabla\times\overline{A}+\sigma\nabla\psi=\overline{J}_s & \text{(3.13b)} \\[2ex] \nabla\bullet\left(-\sigma\dfrac{d\overline{A}}{dt}\right)+\nabla\bullet(-\sigma\nabla\psi)=0 & \text{(3.14b)} \\[2ex] \nabla\bullet\overline{A}=0 & \text{(3.9)} \\[2ex] \nabla\bullet\overline{J}=0 & \text{(3.5)} \end{cases}$$

In electromagnetic accelerators and their power supplies stranded and transposed conductors are widely used. Such a situation is characterized by the suppression of the eddy currents which would otherwise perturb the quasistationary distribution of the current densities in the conductors. In the system of equations (I), such conductors (shown as subdomains $\Omega_{J_1},\Omega_{J_n}..,\Omega_{J_n}$) are considered as having zero conductivity, since imposing $\sigma=0$ in the numerical code forces the eddy currents J_e to be zero. Then,

$$J_s = J_s^* \qquad (3.5)$$

is prescribed as a function of time, for example, and the condition (1.14d) becomes:

$$\nabla\bullet J_s^*=0 \qquad (3.5a)$$

Using circuit analysis methods, J_s^* can be obtained with good approximation.

3.5 THE FINITE ELEMENT FORMULATION FOR THE 3-D, TRANSIENT ELECTROMAGNETIC PROBLEM USING GALERKIN METHOD

The Galerkin method [33], [34], [10] permits one to obtain an approximate solution to a differential equation. In the finite element method, the application of Galerkin technique is based on the requirement of orthogonality between the functions used for approximation and the error between the approximate and the exact solution to the differential equation. Considering a differential equation $Lu - f = 0$ (where L is a linear differential operator) which has an approximate solution $\bar{u} = \sum N_i u_i$, then the error (or residual) $\varepsilon = L\bar{u} - f$ can be efficiently minimized if we require the annulment of integrals $\int_D N_i \varepsilon dw = 0$ for each of the basis functions N_i.

These sets of integrals state mathematically that the basis functions must be orthogonal to the error over the entire domain D.

The finite element procedure is deeply rooted in variational calculus, the solutions being obtained by minimizing a functional over the domain under consideration. The Galerkin method permits that finite element formulations to be extended to domains where no variational principle is feasible. The transient electromagnetic problem of diffusion of fields is one of such problems which being described by parabolic differential equations with partial derivatives refer, to lossy systems.

The finite element formulation for the 3-D problem looks at first sight, similar to the formulation used for the much simpler problem of two-

dimensional axisymmetric diffusion (Pillsbury [24]). The simplicity of the last one is assured by the need to find <u>only one component</u> for the vector potential $\left[\bar{A} = \bar{A}_\theta\right]$ while in the three-dimensional problem which is the object of the present work it is necessary to find <u>all three components</u> of the vector potential $\left[\bar{A} = \bar{A}_x + \bar{A}_y + \bar{A}_z\right]$.

As in the simpler problem to the unknowns ($\bar{A}$=vector potential, Ψ=scalar potential, and λ=Lagrange multiplier), we associate a set of test functions $\bar{u}$, v, and τ, respectively.

Among the conditions required in applying the Galerkin method are:

a. The test functions must be chosen from <u>the same family</u> as the trial functions;

b. The test and the trial functions must be <u>linearly independent</u>;

c. The trail functions should satisfy the boundary conditions and initial conditions <u>exactly</u>.

Condition a) defines the Galerkin method, condition b) is necessasry to ensure that independent equations are available to obtain the unknown coefficients; and, finally, condition c) relates to the efficiency of the method.

In order to apply the Galerkin method, the dot product of the test functions with the governing equations is integrated over the volume (w) of each element and summated over all k elements.

$$\sum_k \left\{ \int_{D_k} \left(\sigma \dot{\bar{A}} + \nabla \times \frac{1}{\mu} \nabla \times \bar{A} + \sigma \nabla \psi - \bar{J}_s \right) \bullet \bar{u} dw + \int_{D_k} \lambda (\nabla \bullet \bar{u}) dw + \int_{D_k} \tau (\nabla \bullet \bar{A}) dw \right\} = 0$$

$$\forall \bar{u}, \tau$$

$$\sum_k \left\{ \int_{D_k} \left[\nabla \bullet \left(\sigma \dot{\bar{A}} \right) + \nabla \bullet (-\sigma \nabla \psi) \right] v dw \right\} = 0$$

$$\forall v$$

Using a vector identity, the element equation is transformed, yielding expressions which contain also the contributions over the boundary of the elements SD_k:

$$\int_{D_k} \nabla \times \left(\frac{1}{\mu} \nabla \times \bar{A} \right) \bullet \bar{u} dw = \int_{D_k} \nabla \bullet \left(\frac{1}{\mu} \nabla \times \bar{A} \right) \times \bar{u} dw + \int_{D_k} \left[\left(\frac{1}{\mu} \nabla \times \bar{A} \right) \bullet (\nabla \times \bar{u}) \right] dw =$$

$$\int_{D_k} \left(\frac{1}{\mu} \nabla \times \bar{A} \right) \bullet (\nabla \times \bar{u}) dw + \int_{SD_k} \left(\frac{1}{\mu} \nabla \times \bar{A} \times \bar{u} \right) \bullet \bar{n} ds =$$

$$\int_{D_k} \left(\frac{1}{\mu} \nabla \times \bar{A} \right) \bullet (\nabla \times \bar{u}) dw + \int_{SD_k} \left(\bar{H} \times \bar{u} \right) \bullet \bar{n} ds =$$

$$\int_{D_k} \left(\frac{1}{\mu} \nabla \times \bar{A} \right) \bullet (\nabla \times \bar{u}) dw + \int_{SD_k} \left(\bar{n} \times \bar{H} \right) \bullet \bar{u} ds =$$

where the integrals over D_k, are volume integrals and over SD_k, are surface integrals, respectively.

In the same manner:

$$\int_{D_k} \left[\nabla \bullet \left(-\sigma \dot{\bar{A}} \right) \right] v dw = \int_{D_k} \left\{ \nabla \bullet v \left(-\sigma \dot{\bar{A}} \right) - \nabla \bullet v \left(-\sigma \dot{\bar{A}} \right) \right\} dw =$$

$$\int_{D_k}\left(\sigma\dot{\bar{A}}\right)\bullet(\nabla v)dw-\int_{SD_k}v\left(-\sigma\dot{\bar{A}}\right)\bullet\bar{n}ds=$$

and also:

$$\int_{D_k}[\nabla\bullet(-\sigma\nabla\psi)]vdw=$$

$$\int_{D_k}(\sigma\nabla\psi)(\nabla v)dw+\int_{SD_k}v(-\sigma\nabla\psi)\bar{n}ds$$

If the integrals referring to the test function v are summarized, their physical meaning becomes evident:

$$\int_{D_k}\left[\nabla\bullet\left(-\sigma\dot{\bar{A}}\right)+\nabla\bullet(-\sigma\nabla\psi)\right]vdw=$$

$$\int_{D_k}\left[\left(\sigma\dot{\bar{A}}\right)\bullet(\nabla v)+(\sigma\nabla\psi)\bullet(\nabla v)\right]dw+\int_{SD_k}v\left(-\sigma\dot{\bar{A}}-\sigma\nabla\psi\right)\bar{n}ds=$$

$$\int_{D_k}\sigma\left[\dot{\bar{A}}+\nabla\psi\right]\nabla vdw+\int_{SD_k}v\bar{J_e}\bullet\bar{n}ds=$$

$$\int_{D_k}-\bar{J_e}\nabla vdw+\int_{SD_k}\bar{J_e}\bullet\bar{n}vds=0$$

$$\forall v$$

Finally:

$$\int_{D_k}\left[\sigma\dot{\bar{A}}\bullet\bar{u}+\left(\frac{1}{\mu}\nabla\times\bar{A}\right)\bullet(\nabla\times\bar{u})+\sigma\nabla\psi\bullet\bar{u}-\bar{J_s}\bullet\bar{u}+\lambda(\nabla\bullet\bar{u})+\tau\left(\nabla\bullet\bar{A}\right)\right]dw+$$

$$\int_{SD_k}\left(\bar{n}\times\bar{H}\right)\bullet\bar{u}ds=0$$

$$\forall\bar{u},\tau$$

and

$$\int_{D_k} \sigma\left[\dot{\overline{A}} + \nabla\psi\right] \bullet \nabla v dw + \int_{SD_k} \overline{J}_e \bullet \overline{n} v ds = 0$$

$$\forall v.$$

The trial functions representing the magnetic vector potential $\overline{A}$, the electrokinetic scalar potential ψ, the Lagrange multipliers λ and the corresponding test functions u,v, and τ must be smooth enough in order to assure that the integrand in the Galerkin formulation is finite. If:

$$\overline{A}, \psi, \overline{u}, v \in \mathbf{H}^1$$

and

$$\lambda, \tau \in L^2$$

Then the weak form obtained by using the Galerkin technique leads to the finite element equations written on element basis:

$$\sum_k^1 \int_{D_k} \left[\sigma \dot{\overline{A}} \bullet \overline{u} - \frac{1}{\mu} \nabla \times \overline{A} \bullet \nabla \times \overline{u} + \sigma \nabla \psi \bullet \overline{u} - \overline{J}_s \overline{u} + \lambda(\nabla \bullet \overline{u}) + \tau\left(\nabla \bullet \overline{A}\right) \right] dw +$$

$$\sum_k^1 \int_{SD_k - SD} \left[\overline{n}_1 \times \overline{H}_1 \bullet \overline{n}_1 + \overline{n}_2 \overline{H}_2 \bullet \overline{n}_2\right] ds + \int_{SD} \overline{n} \times \overline{H} \bullet \overline{n} ds = 0$$

$$\forall \overline{u}, \tau$$

For electromagnetic accelerators and their power supplies, a minimum electromagnetic signature represents a figure of merit.

Then, prescribing a zero value for the potential vector at far-field ($R = \infty$) represents a normal condition and occurrence. The values of the fields become negligibly small at relatively short distances because of compensations and efficiency considerations. It follows that at the boundary SD of the domain being analyzed, $\bar{u} = 0$, and this means that:

$$\int_{SD} (\bar{n} \times \bar{H}) \bullet \bar{u} ds = 0.$$

The assumption was made that $\bar{u} \in \mathbf{H}^1$, implying that $\bar{u}$ is continuous across the interface, then:

$$\sum_k \int_{SD_k - SD} [\bar{n}_1 \times \bar{H}_1 \bullet \bar{u}_1 + \bar{n}_2 \times \bar{H}_2 \bullet \bar{u}_2] ds =$$

$$\sum_k \int_{SD_k - SD} [\bar{n}_1 \times (\bar{H}_1 - \bar{H}_2)] \bullet \bar{u} ds = 0$$

since

$$\bar{n}_1 \times \bar{H}_1 = \bar{n}_1 \times \bar{H}_2$$

As a result, the finite element equations, written on the element basis using a Galerkin technique, simplifies to:

$$\sum_k \left\{ \begin{array}{l} \int_{D_k} \left[\sigma \dot{\bar{A}} \bullet \bar{u} + \frac{1}{\mu} \nabla \times \bar{A} \bullet \nabla \times \bar{u} + \sigma \nabla \psi \bullet \bar{u} - \bar{J} \bullet \bar{u} \right] dw + \\ \int_{D_k} [-\lambda (\nabla \bullet \bar{u}) + \tau (\nabla \bullet \bar{A})] dw \end{array} \right\} = 0 \tag{3.16}$$

The "electrokinetic" problem considerations lead to the finite element equations written on element basis:

$$\sum_k \int_{D_k} \sigma\left[\dot{\bar{A}} - \nabla\psi\right] \bullet \nabla v dw + \sum \int_{SD_k - SD} \left[\bar{J}_{e_1} \bullet \bar{n}_1 v_1 + \bar{J}_{e_2} \bullet \bar{n}_2 v_2\right] ds + \int_{SD} \bar{J}_e \bullet \bar{n} v ds = 0$$

$$\forall v$$

Using the same far-field considerations as above, we assume with sufficient accuracy that the scalar (electrokinetic) potential is zero at the boundaries of the domain (v=0 on SD). Then,

$$\int_{SD} \bar{J}_e \bullet \bar{n} v ds = 0$$

By virtue of the assumptions made before, $\bar{u} \in \mathbf{H}^1$, v is continuous at the interfaces; then:

$$\sum \int_{SD_k - SD} \left[\left(\bar{J}_{e_1} \bullet n_1\right) v_1 + \left(\bar{J}_{e_2} \bullet n_2\right) v_2\right] ds =$$

$$\sum \int_{SD_k - SD} \left(\bar{J}_{e_1} - \bar{J}_{e_2}\right) \bullet n_1 v_1 ds = 0$$

since $\bar{J}_e \bullet \bar{n}_1 = \bar{J}_{e_2} \bar{n}_2$.

then:

$$\sum_k \int_{D_k} \left[\sigma \dot{\bar{A}} \bullet \nabla v + \sigma \nabla\psi \bullet \nabla v\right] dw = 0 \tag{3.17}$$

Relations (3.16) and (3.17) represent the final form of the system of equations on element basis for the Galerkin approach of the 3-D transient finite element code.

In actually solving a particular electromagnetic problem, it is important to systematically look at the form taken by the boundary conditions for *H*, *B*, and *J*:

$$\bar{n} \times \bar{H} = 0;$$
$$\bar{J} \bullet \bar{n} = 0;$$

$$\bar{B} \bullet \bar{n} = 0;$$
$$\bar{n} \times \bar{J} = 0;$$

For the potentials (vector and scalar) $\bar{A}$ and ψ, a similar systematic look is necessary:

$$\nabla \bullet \left(\bar{n} \times \bar{A}\right) = A \bullet \nabla \times \bar{n} - \bar{n} \bullet \nabla \times A = \bar{n} \bullet \bar{B}$$

If $\bar{n} \times \bar{A} = 0$, then $-\bar{B} \bullet \bar{n} = 0$

$$\bar{n} \times \bar{J} = \bar{n} \times \left(\sigma \dot{\bar{A}} - \sigma \nabla \psi\right) =$$

$$-\sigma\left[\bar{n} \times \dot{\bar{A}} + \bar{n} \times \nabla \psi\right] = \bar{n} \times \nabla \psi$$

For nonconductive materials, as well as for stranded and transposed conductors, σ is considered zero:

$\sigma = 0$ $\qquad\qquad\qquad\qquad$ $\bar{n} \times \bar{J} = 0$

For other conductive materials $\sigma \neq 0$, but since
$\psi = \psi^i =$ constant, then $\bar{n} \times \bar{J} = 0$ (where $i = 1, 2..n$) n being the number of conductors.

3.6 FINITE ELEMENT INTERPOLATION

$\bar{A} = A_j\bar{e}_j$

$\bar{u} = u_i\bar{e}_i$

$A_j = N_B A_j^B \qquad \psi = N^\ell \psi_\ell \qquad \lambda = M^n \lambda_n$

$u_i = N_\alpha u_i^\alpha \qquad v = N^k V_k \qquad \tau = M^m \tau_m$

$$* \qquad \sigma \frac{d\bar{A}}{dt} \bullet \bar{u} = \sigma \frac{d\bar{A}_j}{dt} u_j \delta_{ij} = \sigma \delta_{ij} N_\beta \frac{dA_j^\beta}{dt} N\alpha u_i^\alpha = \sigma N_\alpha N_\beta \delta_{ij} \frac{d\bar{A}_j^\beta}{dt} u_i^\alpha$$

$$* \qquad \sigma \nabla \psi \bullet \bar{u} = \sigma \psi_{,j} u_i = \sigma N^\ell_{,i} \psi_\ell u_i = \sigma N^\ell_{,i} N_\alpha \psi_\ell u_i^\alpha$$

$$* \qquad \bar{J} \bullet \bar{u} = J_i N_\alpha u_i^\alpha$$

$$
\begin{aligned}
* \qquad \frac{1}{\mu} \nabla \times \dot{\bar{A}} \bullet \nabla \times \bar{u} \qquad &= \frac{1}{\mu} e_{pqr} A_{r,q} e_{pst} u_{t,s} \\
&= \frac{1}{\mu} \left(\delta_{qs} \delta_{rt} - \delta_{qt} \delta_{rs} \right) A_{r,q} u_{t,s} \\
&= \frac{1}{\mu} \left(A_{t,s} u_{t,s} - A_{s,t} u_{t,s} \right) \\
&= \frac{1}{\mu} \left(N_{\beta,s} A_t^\beta N_{\alpha,s} u_t^\alpha - N_{\beta,t} A_s^\beta N_{\alpha,s} u_t^\alpha \right) \\
&= \frac{1}{\mu} \left(N_{\beta,s} N_{\alpha,s} A_t^\beta u_t^\alpha - N_{\alpha,s} N_{\beta,t} A_s^\beta u_t^\alpha \right) \\
&= \frac{1}{\mu} \left(N_{\beta,s} N_{\alpha s} \delta_{ij} - N_{\alpha,j} N_{\beta,i} \right) A_j^\beta u_i^\alpha
\end{aligned}
$$

where e_{rst} is the permutation symbol defined by the equation:

$e_{111}=e_{222}=e_{333}=e_{112}=e_{113}=....=0$

$e_{123}=e_{231}=e_{321}=1$

$e_{213}=e_{321}=e_{132}=-1$

equal to zero whenever the values of any two indices coincide; e_{ikj} equals 1 when the subscripts permute like 1,2,3, and e_{ijk}=-1 otherwise.

If δ is the Kronecker symbol, e relates to δ according to the identity:

$$e_{ijk}e_{ist} = \delta_{js}\delta_{kt}\delta_{jt}\delta_{ks}$$

* $$\lambda(\nabla \bullet \overline{u}) = \lambda\, u_{i,i} = M_n\, \lambda_n N_{\alpha,i} u_i^\alpha = M^n N_{\alpha,i} \lambda_n u_i^\alpha$$

* $$\tau(\nabla \bullet \overline{A}) = \tau A_{j,j} = M^m N_{\beta,j} \tau^M A_j^\beta$$

3.7 ELEMENT EQUATIONS

$$\left\{\begin{matrix} \left[\int\limits_{D_k} \sigma N_\alpha N_\beta \delta_{ij}\right]\dfrac{dA_j^\beta}{dt} + \left[\int\limits_{Dk}\dfrac{1}{\mu}\left(N_{\beta,s}N_{\alpha,s}\delta_{ij} - N_{\alpha,j}N_{\beta i}\right)\right]A_j^\beta + \\ \left[\int\limits_{D_k} \sigma N_\alpha N_i^\beta\right]\psi_\ell + \left[\int\limits_{D_k} M^n N_{\alpha,i}\right]\lambda_n - \int\limits_{D_k} J_i N_\alpha \end{matrix}\right\} u_i^\alpha +$$

$$+\left\{\int\limits_{D_k} M^m N_{\ell,j} A_j^\beta\right\}\tau^m = 0$$

$$\forall u^\alpha, \tau^m$$

$$\left[\int\limits_{D_k}\left(\sigma N_\beta N_j^\beta \frac{dA_j^\beta}{dt} + \sigma N_{,s}^\ell N_{,s}^k \psi_{\ell,}\right)\right] v_k = 0$$

$$\forall v_k$$

Then:

$$* \quad \left[\int_{D_k} \sigma N_\alpha N_\beta \delta_{ij}\right]\frac{dA_j^\beta}{dt} + \left[\int_{D_k} \frac{1}{\mu}\left(N_{\beta,s}N_{\alpha,s}\delta_{ij} - N_{\alpha,j}N_{\beta i}\right)\right]A_j^\beta + \left[\int_{D_k} \sigma N_\alpha N_i^\ell\right]\psi_\ell +$$

$$\left[\int_{D_k} M^n N_{\alpha,i}\right]\lambda_n = \int_{D_k} J_i N_\alpha$$

where $i = 1,2,3$ and $\alpha = 1,2,...N$, N being the number of nodes.

$$* \quad \sigma\frac{d\tilde{A}}{dt}\nabla v = \sigma\frac{dA_j}{dt}v_{,j} = \sigma N_\beta \frac{dA_j^\beta}{dt}N^\beta v_k = \sigma N_\beta N_{,j}^k \frac{dA_j^\beta}{dt}v_k$$

$$* \quad \sigma\nabla\psi\,\nabla v = \sigma\psi_{,s}v_{,s} = \sigma N_s^\ell \psi_\ell N_{,s}^k v_k = \sigma N_{,s}^\ell N_s^k \psi_\ell v_k$$

$$\left[\int_{D_k} M^m N_{\beta,j}\right]A_j^\beta = 0 \qquad m = 1,2,\ldots M$$

where M is the number of multipliers

$$*\left[\int_{D_k} \sigma N_\beta N_{,j}^\beta\right]\frac{dA_j^\beta}{dt} + \left[\int_{D_k} \sigma N_{,s}^\ell N_{,s}\right]\psi_\ell = 0$$

We denote:

$$\underset{\sim}{x} = \begin{vmatrix} A_j^\beta \\ \psi_\ell \\ \lambda_n \end{vmatrix} \qquad \underset{\sim}{x} = \begin{vmatrix} \frac{dA_j^\beta}{dt} \\ \frac{d\psi_\ell}{dt} \\ \frac{d\lambda_n}{dt} \end{vmatrix} \tag{3.18}$$

Then, the system of equations in compact matrix form is:

$$\underset{\sim}{C}\,\dot{\underset{\sim}{x}} + \underset{\sim}{k}\,\underset{\sim}{x} = \underset{\sim}{F} \tag{3.19}$$

where

$$
\underset{\sim}{C} = \begin{array}{c} \\ \alpha \\ k \\ m \end{array} \begin{array}{ccc} \beta_j & \ell & n \\ \hline \end{array}
$$

$$
\underset{\sim}{C} = \begin{array}{c|ccc|} & \beta_j & \ell & n \\ \alpha & \int_{D_k} \sigma N_\alpha N_\beta \delta_{ij}, & 0, & 0 \\ k & \int \sigma N_\beta N^k_{,j}, & 0, & 0 \\ m & 0, & 0, & 0 \end{array} \tag{3.20}
$$

$$
\underset{\sim}{k} = \begin{array}{c|ccc|} & \beta_j & \ell & n \\ \alpha & \int_{D_k} \frac{1}{\mu}\left(N_{\ell,s} N_{\alpha,s} \delta_{i,j} - N_{\alpha,j} N_{\beta,i}\right), & \int_{D_k} \sigma N_\alpha N^\ell_i, & \int_{D_k} M^n N_{\alpha,i} \\ k & 0, & \int_{D_k} \alpha N^\ell_s N^k_s, & 0 \\ m & \int_{D_k} M^M N_{\beta,j}, & 0, & 0 \end{array} \tag{3.20a}
$$

$$
\underset{\sim}{F} = \begin{vmatrix} \int_{D_k} J_{i,N} \\ 0 \\ 0 \end{vmatrix} \tag{3.20b}
$$

3.8 TIME MARCHING ALGORITHM

In solving the transient (diffusion) electromagnetic problem described by the finite element system of equations:

$$
\underset{\sim}{C}\,\dot{\underset{\sim}{x}} + \underset{\sim}{k}\,\underset{\sim}{x} = \underset{\sim}{F} \tag{3.19}
$$

the first derivative of the nodal point unknowns is approximated by the discrete form:

$$\underline{x} = (1-\theta)\underline{x}^{n} + \theta\,\underline{x}^{n+1}$$

$$\dot{x} = \frac{\dot{x}^{n+1} - x^{n}}{\Delta t}$$

leading to:

$$\underline{C}\left(\frac{\underline{x}^{n+1} - \underline{x}^{n}}{\Delta t}\right) + \underline{k}\left[(1-\theta)\underline{x}^{n} + \theta\,\underline{x}^{n+1}\right] = (1-\theta)\underline{F}^{n} + \theta\,\underline{F}^{n+1}$$

and further:

$$\left[\frac{\underline{C}}{\Delta t} + \theta\,\underline{k}\right]\underline{x}^{n+1} = \left[\frac{\underline{C}}{\Delta t} - (1-\theta)\underline{k}\right]\underline{x}^{n} + (1-\theta)\underline{F}^{n} + \theta\,\underline{F}^{n+1}$$

when $\theta = 1$ we have a backward difference form of the ordinary differential equations which is unconditionally stable.

The matrix equation becomes:

$$\begin{vmatrix} \dfrac{C^{A}_{\alpha\beta ij}}{\Delta t} + k^{A}_{\alpha\beta ij}, & k^{\psi}_{\alpha i\ell}, & k^{\lambda}_{\alpha in} \\ C^{\psi}_{\beta jk_{\psi\theta=0}}, & \Delta t k^{\psi}_{\ell,k}, & 0 \\ \theta\;\; k^{\lambda}_{\beta jm}, & 0, & 0 \end{vmatrix} \bullet \begin{vmatrix} A^{\beta}_{j} \\ \psi_{\ell} \\ \lambda_{n} \end{vmatrix}_{(n+1)} =$$

$$\begin{vmatrix} \dfrac{C_{\alpha\beta ij}}{\Delta t}, & 0, & 0 \\ C^{\psi}_{\beta jk}, & 0, & 0 \\ 0, & 0, & 0 \end{vmatrix} \bullet \begin{vmatrix} A^{\beta}_{j} \\ \psi_{\ell} \\ \lambda_{n} \end{vmatrix} = \begin{vmatrix} \int_{D_k} J^{n+1}_{i} N^{\alpha} \\ 0 \\ 0 \end{vmatrix} \qquad (3.21)$$

where

$$
\begin{aligned}
C^{A}_{\alpha\beta ij} &= \int_{D_k} \sigma N_\alpha N_\beta \delta_{ij} \\
C^{\psi}_{\beta jk} &= \int_{D_k} \sigma N_\beta N^{k}_{,j} \\
k^{A}_{\beta ij} &= \int_{D_k} \frac{1}{\mu}\left(N_{\beta,s} N_{\alpha,s} \delta_{ij} - N_{\alpha,j} N_{\beta,i}\right) \\
k^{\psi}_{\alpha i\ell} &= \int_{D_k} \sigma N_\alpha N^{\ell}_{,i} \\
k^{\lambda}_{\alpha in} &= \int_{D_k} M^{n} N_{\alpha,i} \\
k^{\psi}_{\ell k} &= \int_{D_k} \sigma N^{\ell}_{,s} N^{k}_{,s} \\
k^{\lambda}_{\beta jn} &= \int_{D_k} M^{m} N_{\beta,j}
\end{aligned}
\qquad
\begin{vmatrix}
\int_{D_k} \left(\sigma A^{n}_{i} + J^{n+1}_{i}\right) N^{x} \\
\int_{D_k} \sigma A^{n}_{i} N^{\beta}_{,j} \\
0
\end{vmatrix}
\qquad (3.21a)
$$

CHAPTER 4

SUPERFICIAL CURRENTS AS LAGRANGE MULTIPLIERS IN THREE-DIMENSIONAL FINITE ELEMENT METHOD FORMULATIONS FOR ELECTROMAGNETIC PROBLEMS

4.1 INTRODUCTION: THE ENFORCEMENT OF COULOMB'S GAUGE

In conditions of quasistatic approximation $\left(\frac{\partial \overline{D}}{\partial t}=0\right)$ which apply to all electromechanical magnetic flux compression and magnetic flux expansion converters, Coulomb's gauge [31], [32], [10] for the magnetic vector potential is "chosen:"

$$div\overline{A}=0. \tag{4.1}$$

The gauge "choice" is necessary, since a vector field must be completely characterized by specifying both its curl and its divergence <u>in every point of its domain of definition</u> [25], [31], [32]. While drastically simplifying the governing equations (see Chapter 3), the main reason for introducing is not to simplify them, as stated in [24], but is enforced in order to satisfy the continuity equation in the domain of interest:

$$divJ=0. \tag{4.2}$$

Equation (4.2) is actually a corollary of the Coulomb gauge for the condition that:

$$R^2\left|\overline{A}\right|<\infty \qquad \text{when } R\to\infty \tag{4.3}$$

since, by taking the divergence of the two members of the Helmholtz equation for magnetic fields (simplified for the magnetostatic conditions [25], [31]):

$$\nabla^2 \overline{A} = -\mu \bullet \overline{J}, \tag{4.4}$$

and considering (4.2), we find that $div\overline{A}$ is a constant and, since its value is zero at the infinity, the constant is zero everywhere,

The enforcement of the Coulomb gauge will mean that, in electromechanical converters, all the conduction currents are closed in themselves, and the energy spent in electromagnetic radiation is negligibly small, even for the case of rapid launchers.

For electromagnetic systems modelling, there exist several two-dimensional f.e.m. numerical codes. Their usefulness is based on the fact that many electromagnetic problems can be simplified to two-dimensional symmetries and solved with more modest computational means.

In such codes, the equation (4.2) and the Coulomb gauge are easily enforced. In an axisymmetric structure, a coil appears as its cross-section and is implied that the current distribution will be identical in all its other cross-sections.

In three-dimensional structures modelling the true, unsimplified structure of the electromagnetic system, the situation changes, especially in the finite element method since the entire domain is built by volume elements (bricks, pyramids, wedges, etc.) which must match the continuity

equation (4.2) not only in some points of the integration, but along and all over the entire domain. In other words, the special role of enforcing the Coulomb gauge in three-dimensional f.e.m. systems is to bridge a method which is fundamentally discrete with a requirement of pointwise continuity. The "plaster" used to "smooth-out" the discrete 3-D electromagnetic structure is a group of superficial currents flowing through the 3-D element faces, thus producing magnetic fields which impose the constraint $div\overline{A} = 0$. Such superficial currents will be assimilated with Lagrange multipliers.

This chapter will be devoted to outlining the special physical meaning of superficial Lagrange multipliers in three dimensional, finite element formulation of electromagnetic problems, in magnetic flux compression devices.

4.2 THREE-DIMENSIONAL ELEMENTS

Finite element method [22], [28], [29], [30] is based on the replacement of a continuous system (which , in our case, is the domain in which the electromagnetic transient problem must be solved) by an equivalent discrete system, made out of finite elements (volume, or three-dimensional, in our case). The geometry of all the elements must be defined analytically and appropriate interpolation functions N_i must be constructed for each element.

To define the geometry of elements, a set of n points is selected. These points, called geometrical nodes, may sometimes coincide with interpolation nodes. Each element is analytically and uniquely defined in terms of geometrical nodes belonging to that element and its boundary. Two distinct

elements can have common points only on their common boundaries if such boundaries exist; no overlapping is allowed. Figure 11 shows elements as rectangular prisms (linear and quadratic) with 8 and, respectively, 20 nodes. In order to assure the interelement continuity, it is necessary that, along a whole face of an element, the nodal values define a unique variation of the unknown function.

Using three normalized coordinates, we have the following shape functions [27], [30]:

For the linear element (8 nodes):

$$N_i = \frac{1}{8}(1+\xi_0)(1+\eta_0)(1+\zeta_0) \tag{4.5}$$

For the quadratic element (20 nodes):

Corner nodes:

$$N_i = \frac{1}{8}(1+\xi_0)(1+\eta_0)(1+\zeta_0)(\xi_0+\eta_0+\zeta_0-2) \tag{4.6}$$

Mid-size nodes:

$$\xi_i = 0 \eta_i = \pm 1 \zeta_i = \pm 1$$

$$N_i = \frac{1}{4}(1-\xi^2)(a+\eta_0)(1+\zeta_0) \tag{4.7}$$

Figure 12 shows triangular prism elements, linear and quadratic, with 6 and, respectively, 15 nodes. In this case, the shape functions are obvious and found in any book on finite element method [27], [30]. Again, for

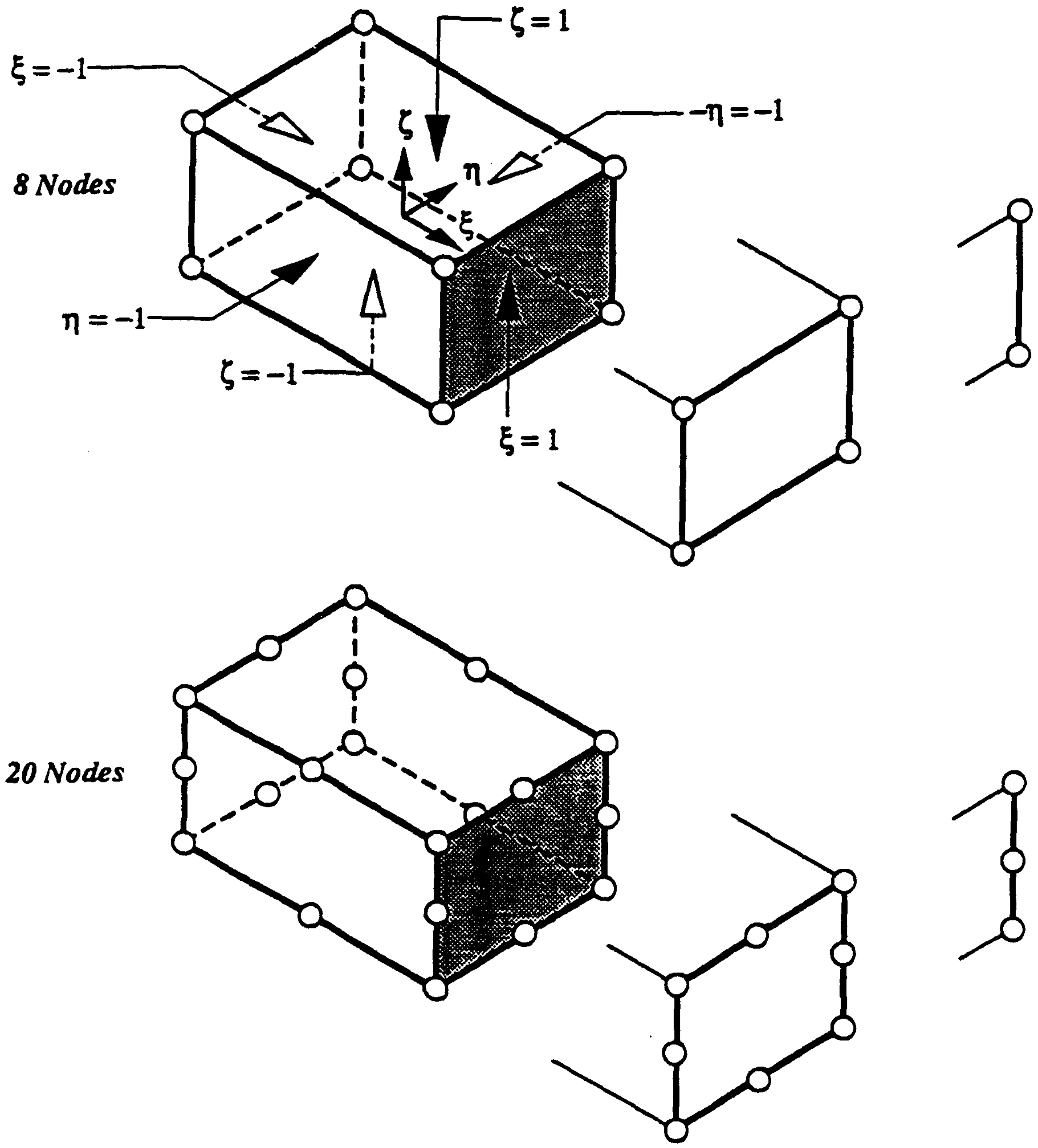


Figure 11. Three-dimensional, finite elements as rectangular prisms with 8 or 20 nodes. Surface currents through faces represent Lagrange multipliers.

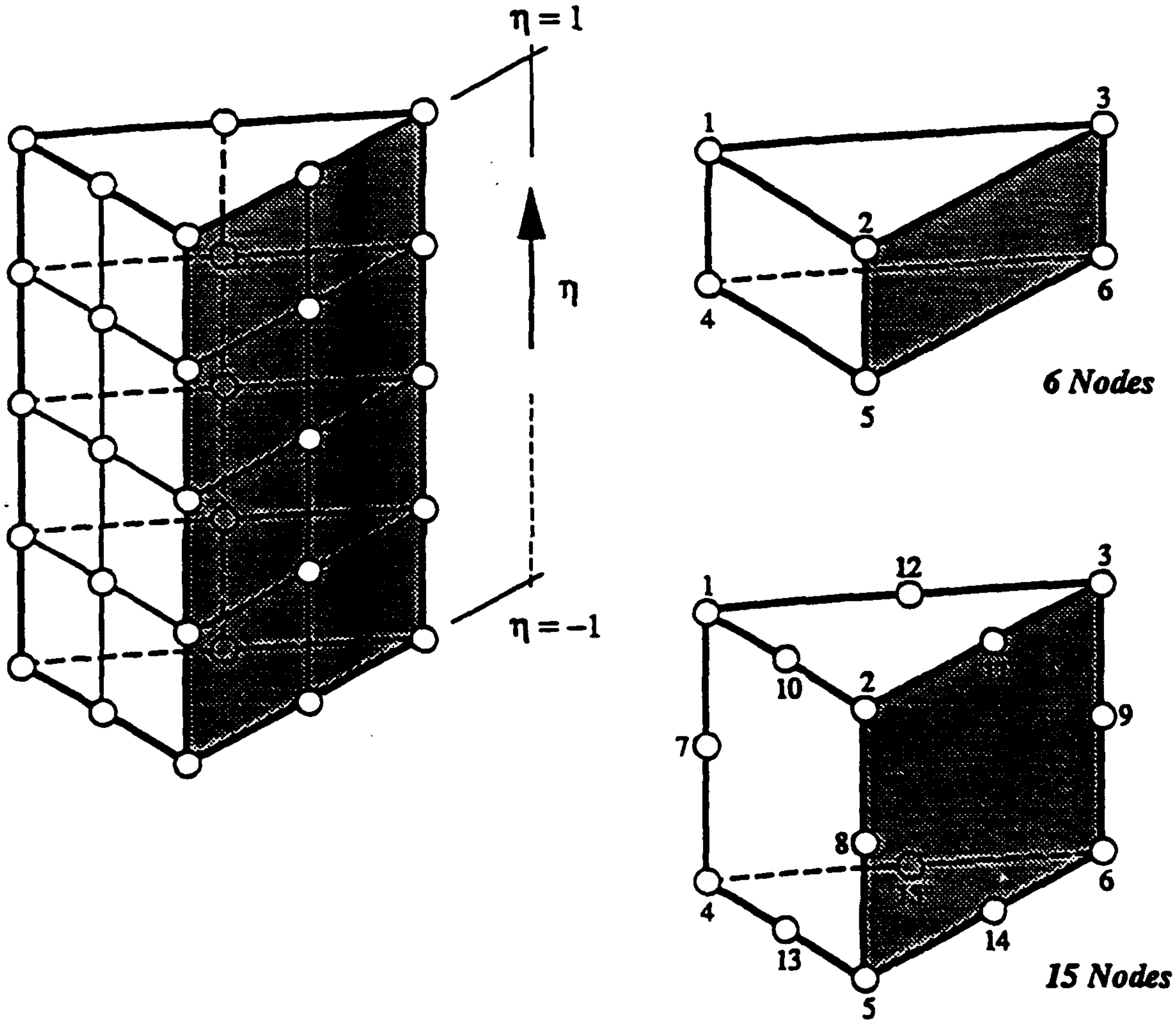


Figure 12. Three-dimensional, finite elements as triangular prism elements with 6 or 15 nodes. Surface currents through faces represent Lagrange multipliers.

interelement continuity, it is necessary that, along a whole face of an element the nodal values define a unique variation of the unknown function.

4.3 PRESCRIPTION OF THE FORCING FUNCTION IN THREE-DIMENSIONAL FINITE ELEMENT ELECTROMAGNETIC PROBLEMS

In electromagnetic problems, the forcing functions are globally, voltage sources or current sources, impressed upon a certain network. In electromagnetic field problems solved by f.e.m. codes, the forcing functions corresponding to voltages are the intensities of applied electric fields and, corresponding to current sources are current densities .

Current densities, for example, are prescribed element by element. Two distinct elements have as common boundaries points, lines, or surfaces. At their common boundaries, the assembled elements should leave no holes or, as mentioned above, overlap. When the boundary of the domain cannot be exactly filled by the elements selected, an error called geometrical discretization error, cannot be avoided.

By prescribing current densities, element by element, the current, representing the surface integral (the flux) of the local current densities, does not obey exactly the first Kirchhoff's law or locally $div\overline{J} = 0$ (or by implication $div\overline{A} = 0$) does not hold, two elements, one hexahedronal, the other prismatic, may be adjacent and the current density prescription as forcing function may lead to discrepancies in the Coulomb's gauge which must be fulfilled pointwise.

Round-off errors in computation of currents - even for an apparently balanced input - lead to discrepancies upsetting the solenoidal character of the current densities.

The method of correcting such discrepancies is the introduction of surface (sheets) currents flowing through the element faces and by the magnetic field they create a compensating effect. Then, the Coulomb's gauge seems to be imposed pointwise in the entire volume of the domain. We are identifying such sheets of currents with Lagrange multipliers.

4.4. PHYSICAL INTERPRETATION OF SUPERFICIAL CURRENTS AS LAGRANGE MULTIPLIERS IN F.E.M. FORMULATIONS OF THREE-DIMENSIONAL ELECTROMAGNETIC PROBLEMS

The Helmholtz theorem [25], [31] states that any vector field is uniquely separable into a divergenceless part, *curl* $\bar{A}$ and a curless part, *grad* ψ, in each point of the field, and in average in each of the three dimensional elements in the finite element formulation. We are interested in the transverse component of the vector field (transverse to the direction of greatest change which is the lamellar or longitudinal one), the Coulomb gauge imposing a linear relationship between the three components of the vector potential $\bar{A}$, decreasing the number of independent components to two.

For the 3-D f.e.m. formulation of the transient electromagnetic problem, the Galerkin method is used in order to obtain the finite element equations. The formulation is carried out in Chapter 3. The equation (3.18) from this chapter is reproduced here:

$$\int_{\pi}\left\{\left[\sigma\frac{\partial \bar{A}}{\partial t}+\nabla\times\frac{1}{\mu}\times\nabla\times\bar{A}+\sigma\nabla\psi-J_m\right]\bullet\bar{u}+\right.$$

$$\left.\lambda(\nabla\bullet\bar{u})+\tau(\nabla\bullet\bar{A})\right\}dv=0 \tag{3.18}$$

for any $\bar{u}$ and τ.

In equation (3.18), the Coulomb gauge constraint div $\bar{A}$ =0 has been enforced using the Lagrange multipliers λ and the corresponding test functions τ.

From the above equations, the dimensionality of the Lagrange multiplier results in $\left[\frac{A}{m}\right]$ and represents a surface current (a current sheet).

In any type of 3-D elements (as in Figure 11, for example), they materialize in the face of the elements. Their use is based on the fact <u>that the magnetic field produced by a distribution of volume currents inside an element</u> (brick, prism, tetrahedron, etc.) <u>can be substituted by an identical magnetic field structure produced by a distribution of surface currents flowing on different faces of the respective element.</u>

The current density for the sheet current, $\bar{K}$ is obtained by passing to limit of a volume current density $\bar{J}$ when one of the dimensions (the thickness t) goes to zero (Figure 13).

The total current I (Figure 14) can be expressed as

$$I=\bar{J}\bullet\bar{n}(lt)=\bar{K}\bullet n_t(\ell)$$

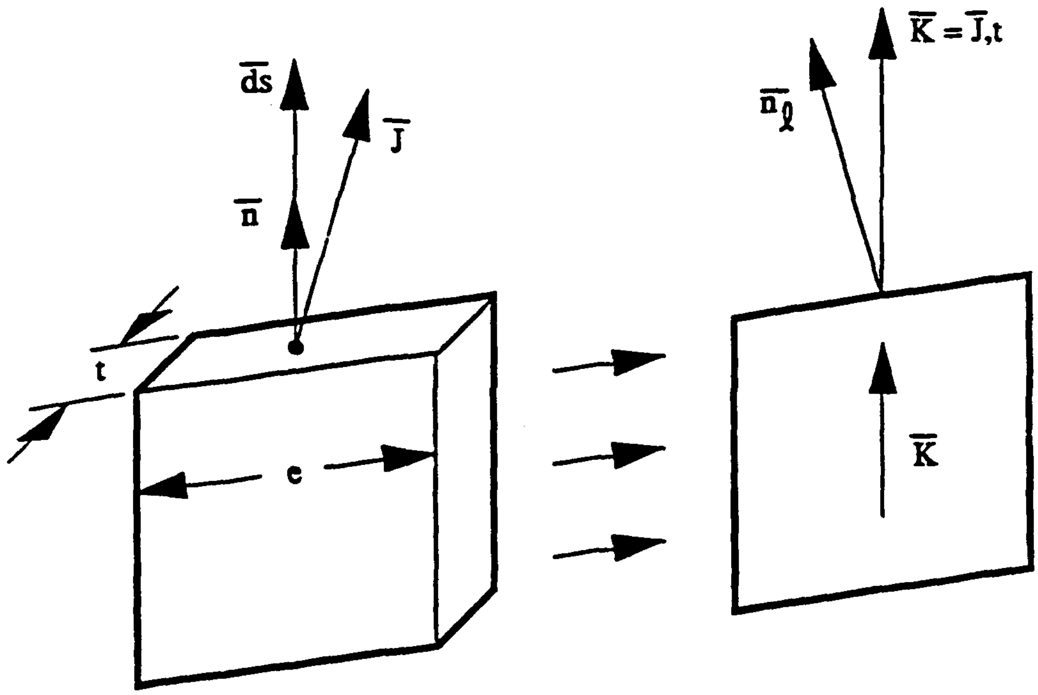


Figure 13. Surface current density K on a face of a 3-D element (brick) as a passing to a limit operation.

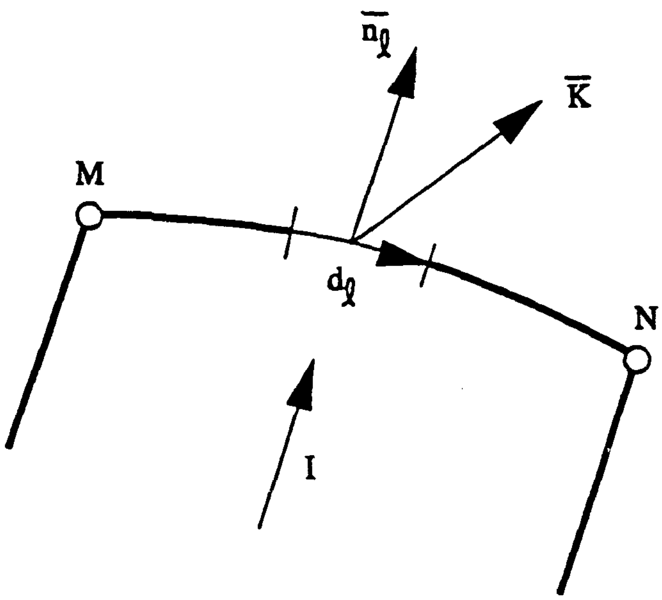


Figure 14. Calculation of current I as a line integral of surface current K.

where

$$\overline{K} = \lim \overline{J} \bullet t \tag{4.8}$$

In general, the surface current I (Figure14) is given by the integral

$$I = \int_M^N \left(\overline{K} \bullet \overline{n}_t\right) d\ell \tag{4.9}$$

The surface currents equivalent of the continuity condition for the volume currents expressed in point form by equation (4.2) is:

$$\oint_c \left(\overline{K} \bullet \overline{n}_t\right) d\ell = 0 \tag{4.10}$$

In some treatments [32], equation (4.8) is replaced by a double integral of a surface divergence of the current density $\overline{K}$.

To illustrate how the Lagrange multipliers act in imposing a constraint (e.g. the gauge condition), let's assume a given distribution of currents $\overline{J}(x,y,z)$ in a volume V already divided in 3-D elements. The volume V is surrounded by the closed surface Σ such that the flux density B_t produced by the volume currents $\overline{J}$ is tangential everywhere to the surface Σ. Adistribution of sheet currents of density $\overline{K}$ over the surface Σ, is chosen (Durand [32]) such that:

$$-\mu_o \overline{K} = \overline{n} \times \overline{B}_t \tag{4.11}$$

Then, multiplying by the positive (outward) normal to Σ, both sides of (4.11) and considering the properties of the double cross product, yields the value of the flux density results as:

$$B_t = \mu_o(\bar{n} \times \bar{K}). \tag{4.12}$$

If we superimpose (considering linear media) the effects of the two current distributions, volume $\bar{J}$ and surface $\bar{K}$ we find the flux densities:

$\bar{B} = \bar{B}_t$ inside of surface Σ

$B=0$ outside of Σ. (4.13)

The known condition of discontinuity (jump) in the value of B when crossing a sheet of currents

$B_{above} - B_{below} = \mu_o[\bar{n} \times \bar{K}]$ is satisfied by the relations (4.12) and (4.13).

From Biot and Savart's law:

$$\bar{B} = \frac{\mu_o}{4\pi} \iint_{\Sigma} \left(\bar{K} \times \frac{\bar{r}}{r^3} \right) ds = \iint_{\Sigma} \left(\bar{K} \times grad\left(\frac{1}{r} \right) \right) ds = \frac{\mu_o}{4\pi} curl \iint_{\Sigma} \frac{\bar{K}}{r} ds \tag{4.14}$$

and knowing both its divergence and its curl $(divB = 0, curl\bar{B} = \mu_o \bar{J})$, $\bar{B}$ is completely defined for the electromagnetic problems concerning electromechanical converters (launchers and electromechanical power supplies.)

If the currents $\bar{K}$ had existed alone, they would have produced outside of surface Σ a flux density equal and opposite to that produced by the volume distribution of currents $\bar{J}$ and a flux density $\bar{B}$ equal to zero inside of Σ.

On this fact lies the physical meaning and the mechanisms of operation of the Lagrange multipliers, acting as current sheets in the 3-D elements in a finite element grid: They are the means to replace the currents $\bar{J}$ for their action outside the boundary Σ by superficial currents flowing over surface Σ.

In the same way, we can consider the case in which the closed surface Σ is exterior to the volume enclosed by the 3-D element (or the set of 3-D elements of which the volume of definition for the electromagnetic problem consists). Following the same reasoning as above, the superficial distribution of the sheet of currents $\bar{K}$ produces inside of boundary Σ the same flux density $B_t t$

$$B_t = -\mu_o[\bar{n} \times \bar{K}] \tag{4.15}$$

as the volume currents $\bar{J}$. Outside of surface Σ, the flux density produced by the current sheet $\bar{K}$ is zero. As an example, in Figure 15 we have a cylinder with a radius r through which a current I of density $J = \frac{I}{\pi r^2}$ flows uniformly.

On the toroidal surface surrounding concentrically the cylinder, a surface current density $\bar{K}$ can always be chosen in order to produce a field B_t inside of the torus, identically equal to zero.

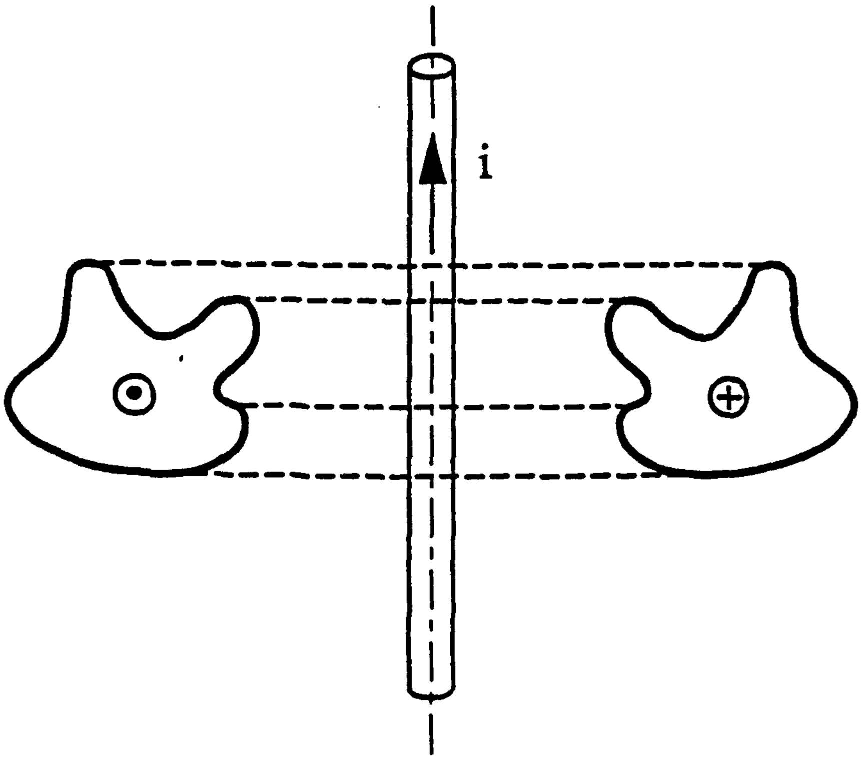


Figure 15. Illustrative of the mechanism of current sheets acting as Lagrange multipliers.

We must emphasize that Lagrange multipliers refer to the vector potential $\overline{A}$, enforcing its divergence to be zero, not to the flux density $\overline{B}$ which is the curl of $\overline{A}$. However, since the topic is the physical significance of the Lagrange multipliers, the flux density which is more familiar was used.

Lagrange multipliers impose the condition *div A=0*. The expression of divergence in terms of surface currents for the magnetostatic problem results from [25], [32]:

$$div\overline{A} = \frac{\mu_o}{4\pi}\iint_{\Sigma}\left(\overline{K}grad\left(\frac{1}{r}\right)\right)ds \tag{4.16}$$

and since $\overline{K}$ is a vector tangent to the surface, we may write equation (4.16) as:

$$div\overline{A} = \frac{\mu_o}{4\pi}\left\{\iint_{\Sigma} div_s\overline{K}\frac{ds}{r} - \iint_{\Sigma} div_s\left(\frac{\overline{K}}{r}\right)ds\right\} \tag{4.17}$$

Where the following vectorial identity was employed:

$$div_s\left(\frac{\overline{K}}{r}\right) = \frac{1}{r}div_s\overline{K} - \left(\overline{K}grad_s\left(\frac{1}{r}\right)\right) \tag{4.18}$$

In (4.17) and (4.18), the *s* index was used in order to indicate the surface gradient and divergence.

Since

$$\iint_{\Sigma} div_s\overline{K}ds = \int_{\Gamma}(\overline{n}\bullet\overline{K})d\ell = 0 \tag{4.19}$$

where the curve Γ surrounds the surface Σ, we conclude that:

$div_s \overline{K} = 0$.

Also, on the curve Γ we have:

$$\left(\overline{n} \bullet \overline{K}\right)_{\Gamma} = 0$$

resulting that $div\overline{A} = 0$ is imposed by surface currents flowing on the faces of elements in electromechanical converters treated here.

The boundary conditions for the vector potential, when crossing a sheet of currents are:

$$\overline{A}_a = A_b$$

and

$$\left(\frac{\partial \overline{A}}{\partial n}\right)_a - \left(\frac{\partial \overline{A}}{\partial n}\right)_b = -\mu_o \overline{K} \tag{4.20}$$

4.5 AN ILLUSTRATATION OF THE USE OF THREE-DIMENSIONAL F.E.M. TRANSIENT ELECTROMAGNETIC CODE

As an illustration of the application of the three-dimensional, finite element transient electromagnetic code using superficial currents as Lagrange multipliers - a simple flux compression structure was chosen. This arrangement comprises a rectangular coil of two turns made of stranded and transposed copper conductor in which a current is impressed in a ramp variation. The current density increases linearly in the coil from 0 to 6.2 x $10^9 \frac{A}{m^2}$ in a time of 25 μsec.

In front of the coil, compressioning the magnetic flux is a rectangular piece of solid copper, having a conductivity of 5.8 x 107 $\frac{1}{\Omega m}$, which is the seat of the eddy currents. Referring to Figure 10 in Chapter 3, the coil is representative of subdomains D_J, the copper plate of subdomains D_E, and the rest of the space, of domains D_O.

The structure is truly three-dimensional; however, it can be grossly approximated by a two-dimensional axisymmetric configuration for a comparison check.

Figures 17 and 18 represent the division of the entire domain considered in the problem in 3-D elements. The electromagnetic (transient) diffusion problem is solved at time intervals of 5 μsec and plotted at intervals of 5, 10, 15, and 20 μsec. The figures represent the magnitude of current density in its vectorial representation and the magnetic field, respectively, in upper face of the copper plate and three-dimensionally at 5, 10, 15, and 20 μsec time.

It is clear that division in a relatively small number of elements takes its toll in the precision of representation. It is also notable that the accuracy remains high, the introduction of Lagrange multipliers playing an important role.

In order to have a term of comparison, a similar, but two-dimensional, axisymmetric system was considered. The elements have comparable size, in spite of differences in shape, the coil is still stranded and tranposed, while the massive copper plate has the same conductivity as its three-dimensional counterpart. The current density was chosen in order that the total magnetomotive force would be comparable for the two coils. The plots are still made for 5, 10, 15, and 20 μsec times.

The axisymmetric structure of the simple flux compressor is given in Figure 2. Figures 3-6 show the unfolding in time of the flux lines as the coil is pulsed with current, and eddy currents are developing in the plate. This two-dimensional case is reproduced from a paper by M. D. Driga and H. D. Fair, "Advanced Concepts for Electromagnetic Launcher Power Supplies Incorporating Magnetic Flux Compression," [35].

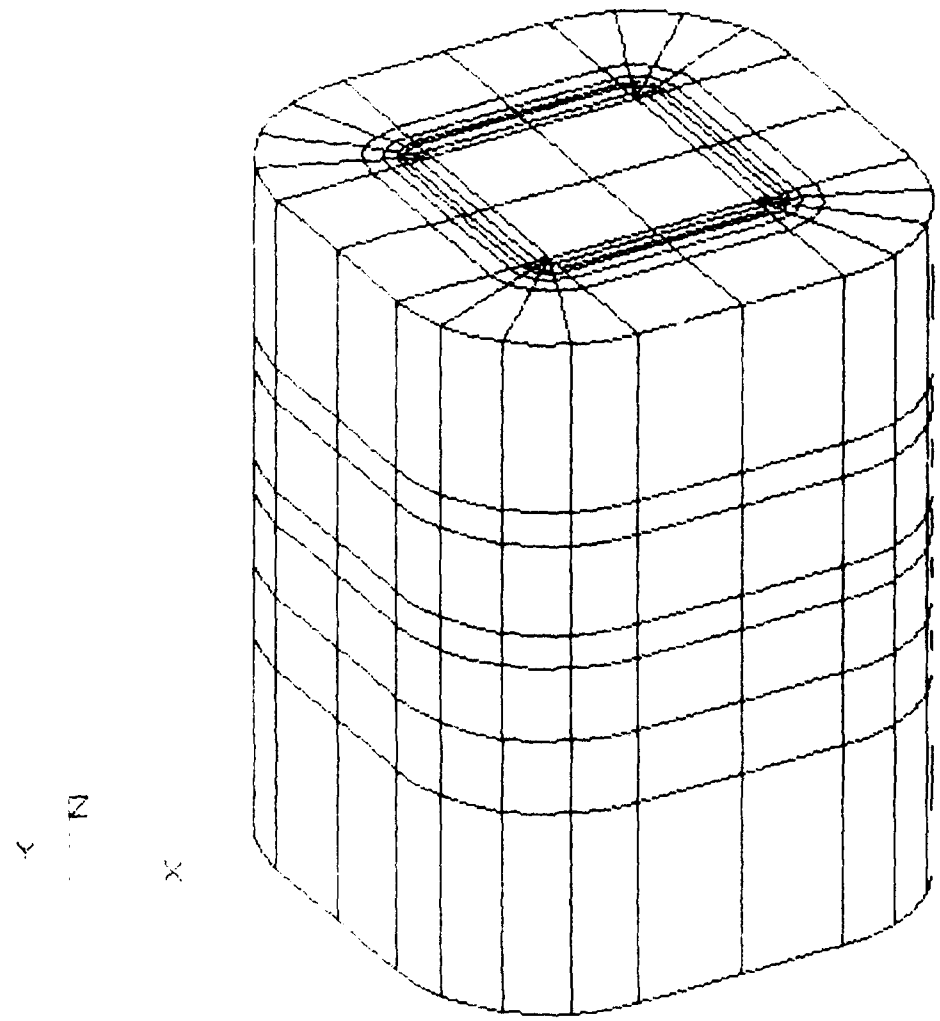


Figure . MFC: domain for electromagnetic transient f.e.m. problem

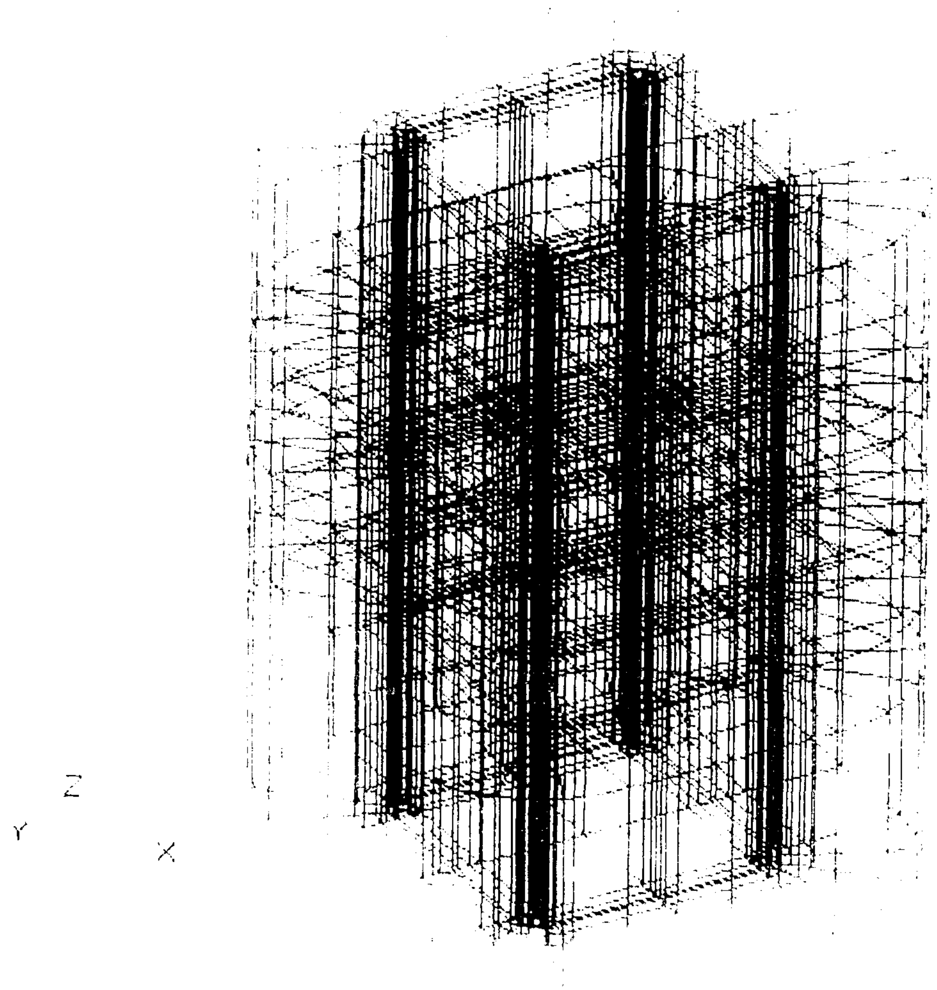


Figure [illegible]: domain division into 3-D elements

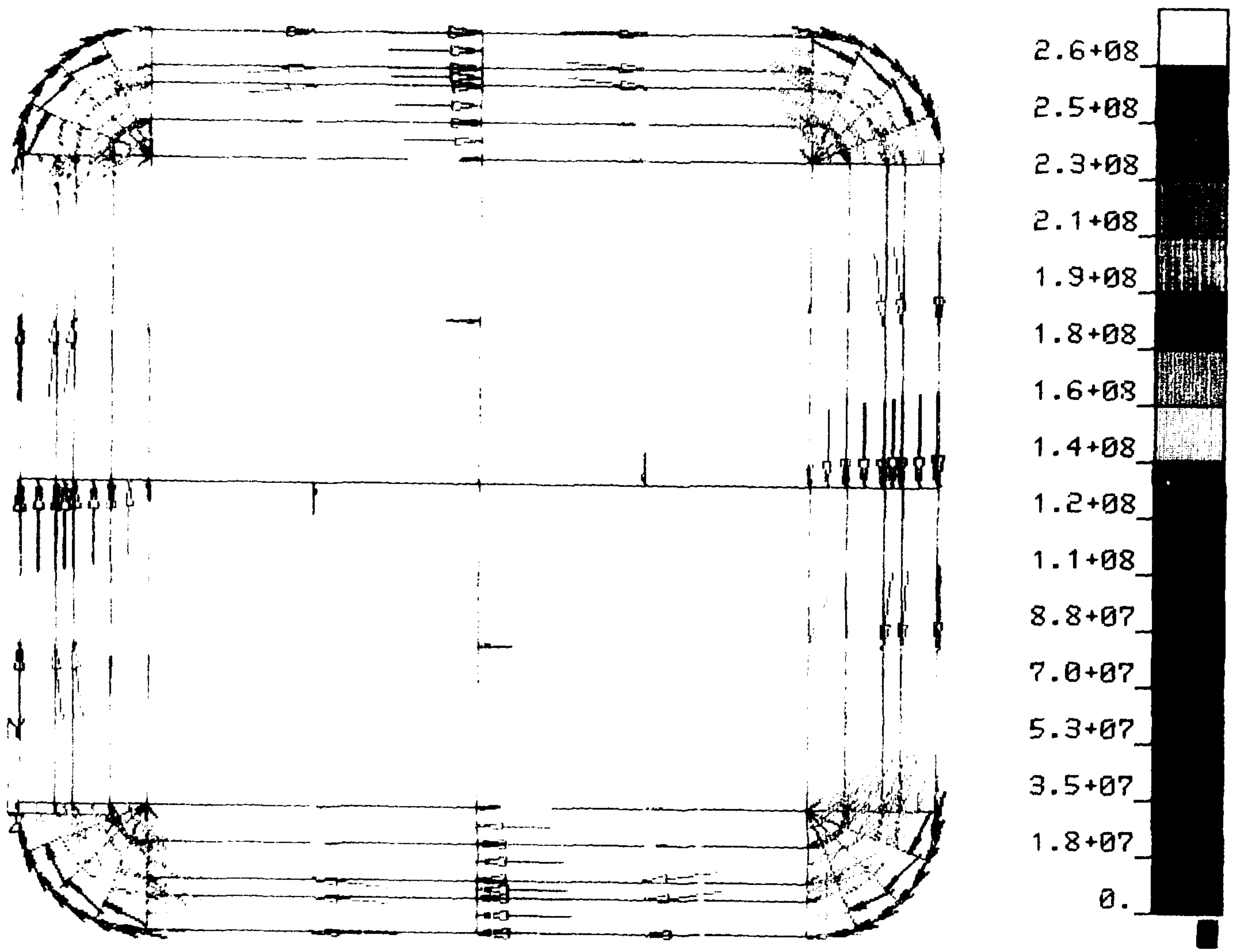


Figure 18. MFC: current density in plate at $t = 5 \mu sec$ (vectorial representation)

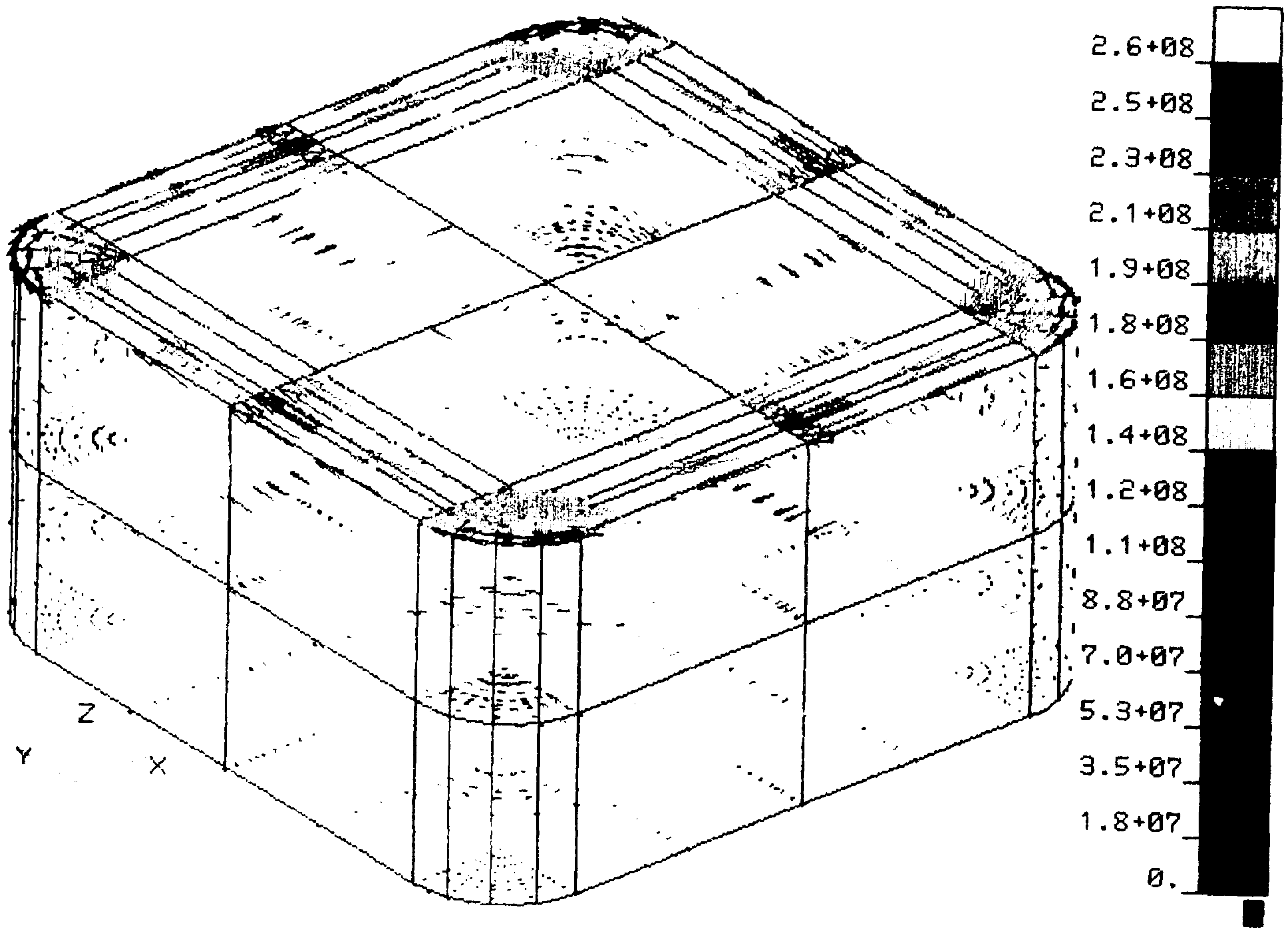


Figure 19. MFC: current density in copper plate at 5 μsec (3-D vectorial representation)

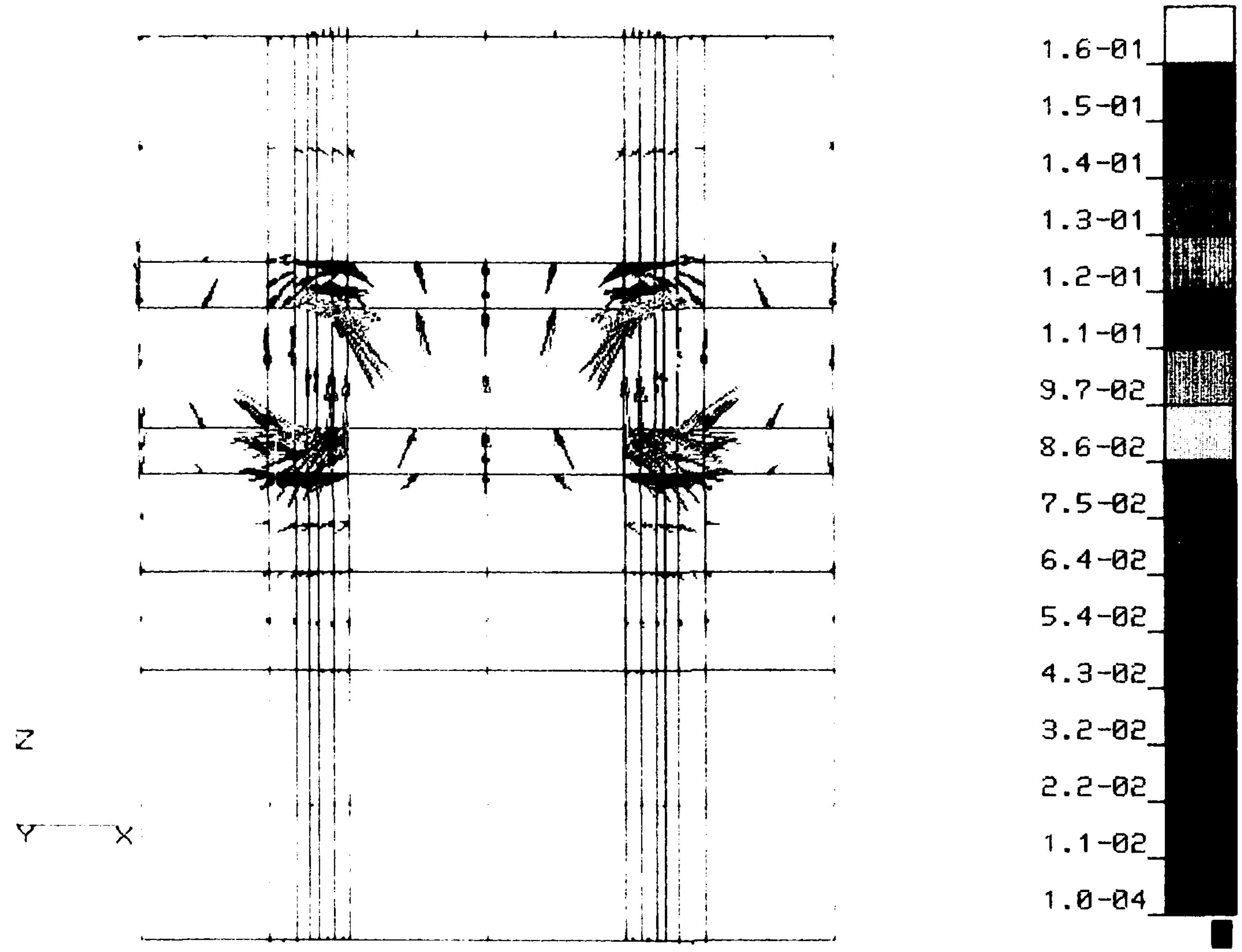


Figure 20. MFC: magnetic flux density at 5μsec (longitudinal section, vectorial representation)

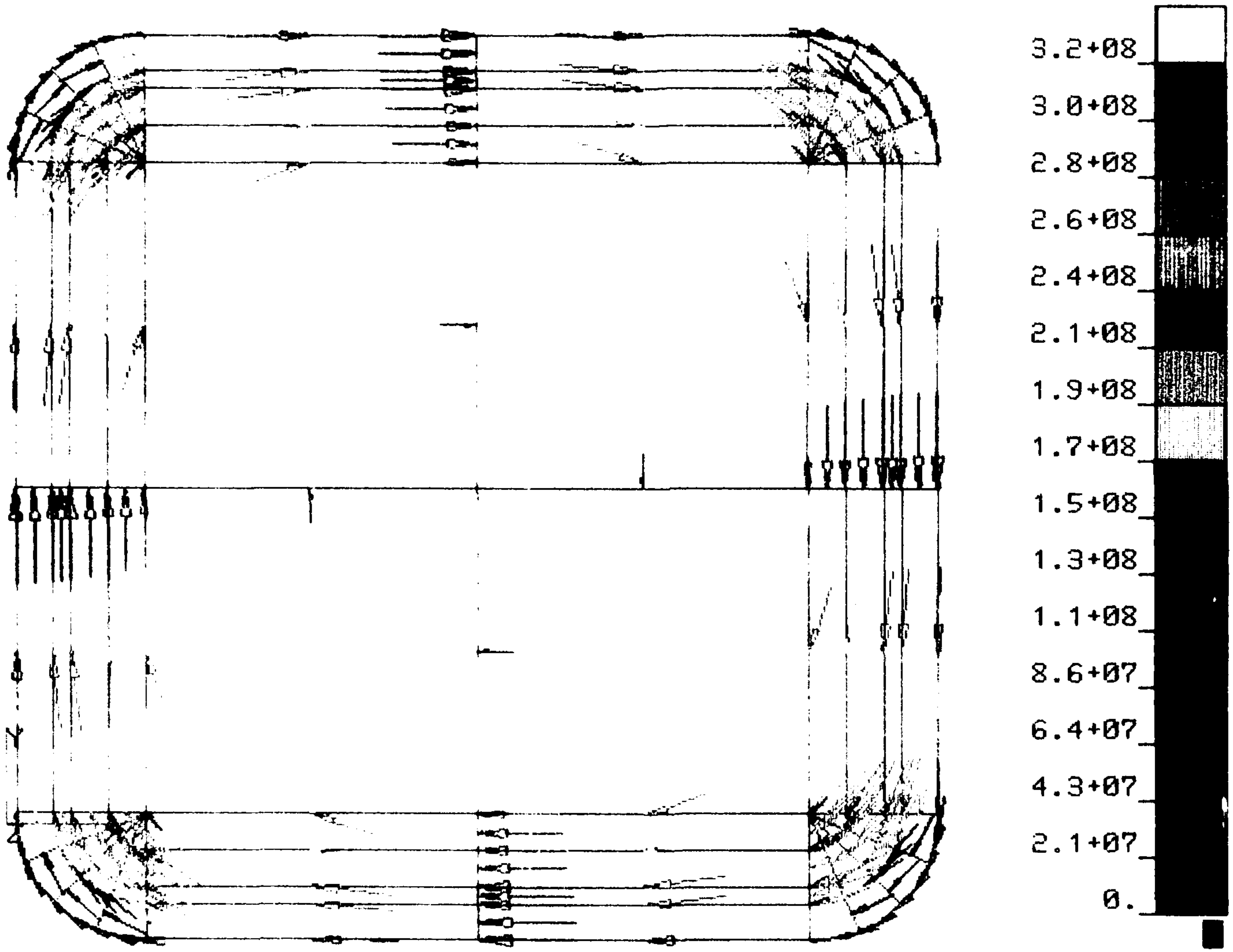


Figure 21. MFC: current density in copper plate at t = 10 μsec (vectorial representation)

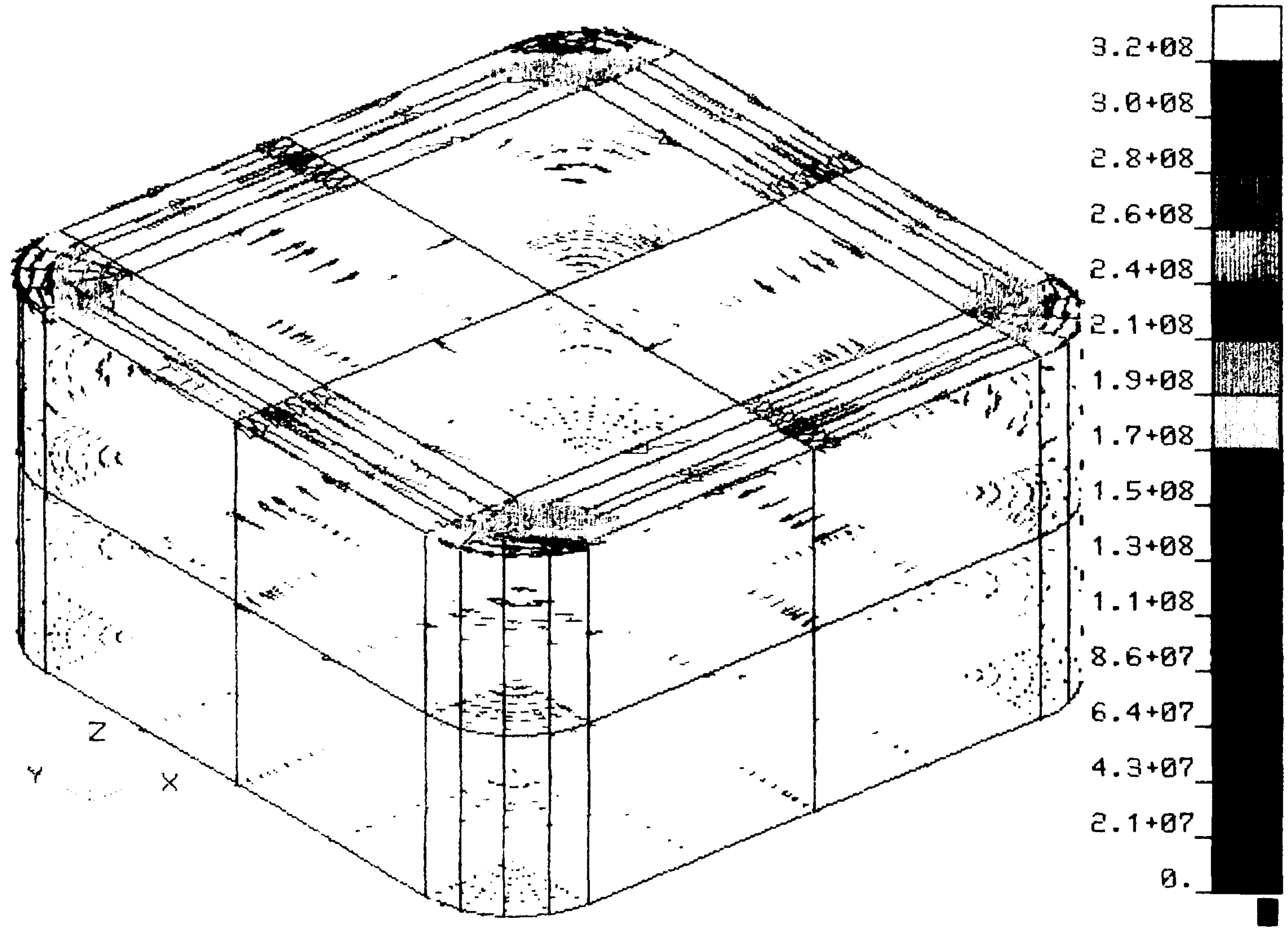


Figure 22 MFC: current density in copper plate at t = 10 μsec (3-D vectorial representation)

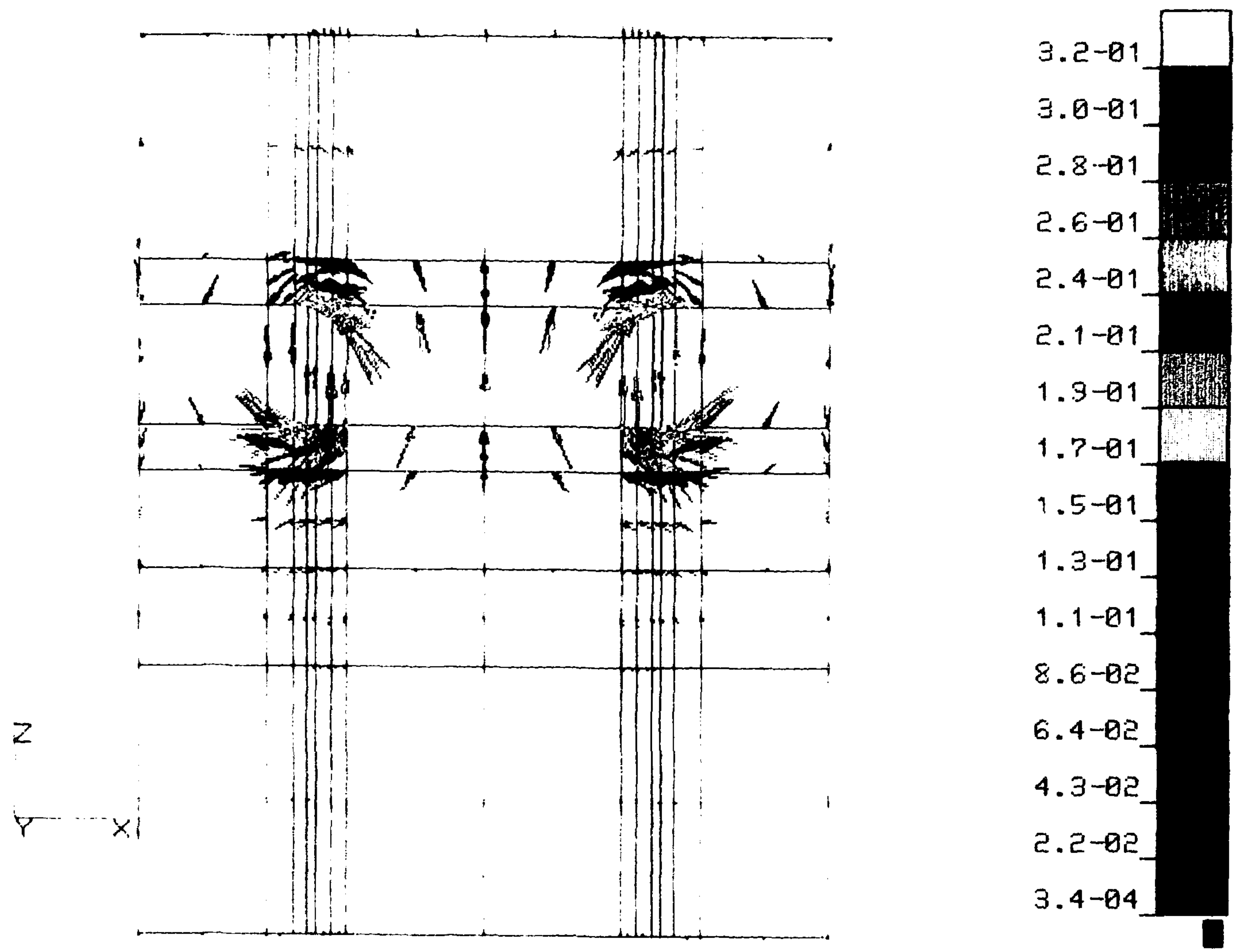


Figure 23 MRC: magnetic flux density at 10μsec (vectorial representation - longitudinal section)

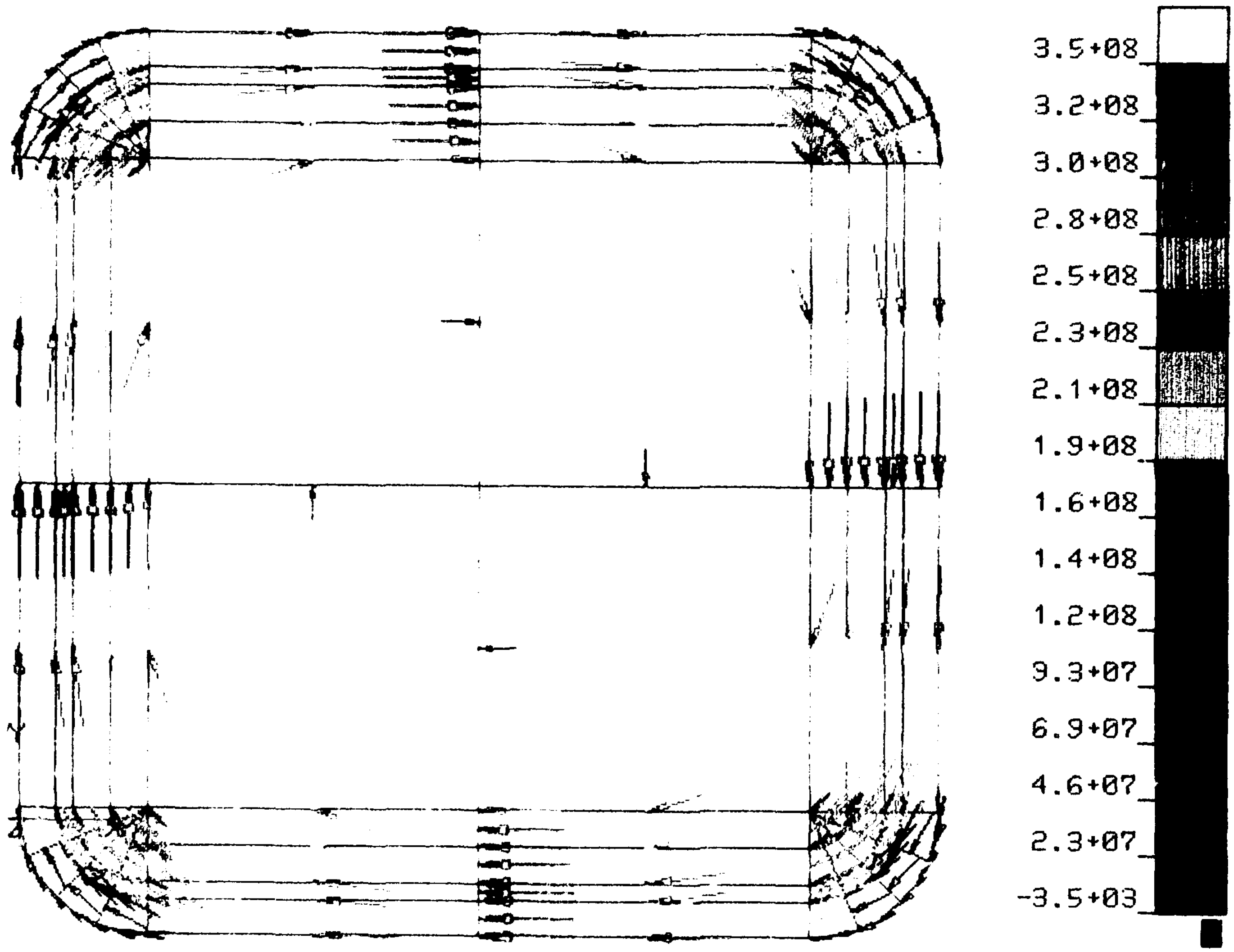


Figure 24. MFC: current density in copper plate at 15 μsec (vectorial representation)

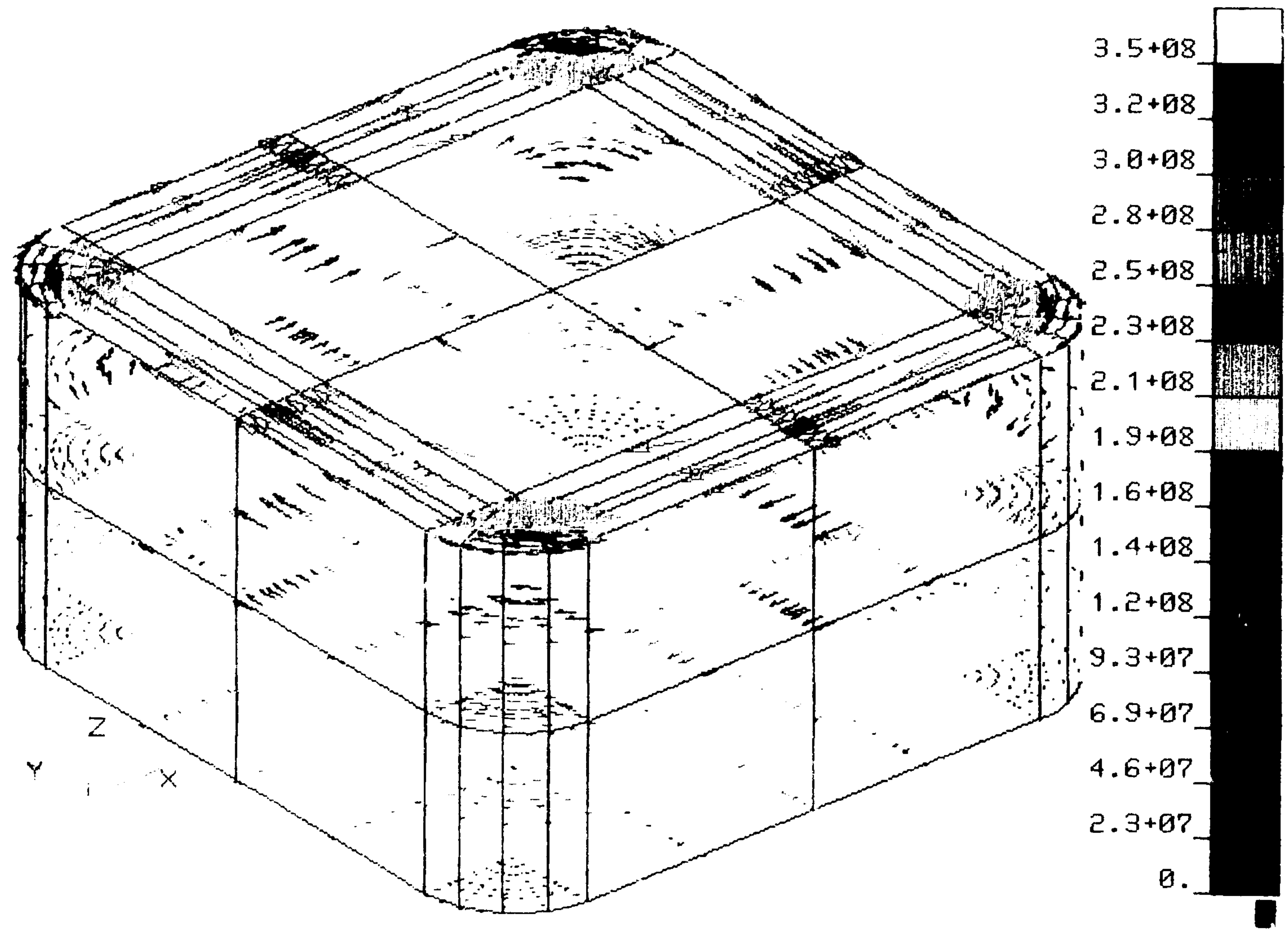


Figure 25. MFC: current density in copper plate at $t = 15\,\mu sec$ (3-D vectorial representation)

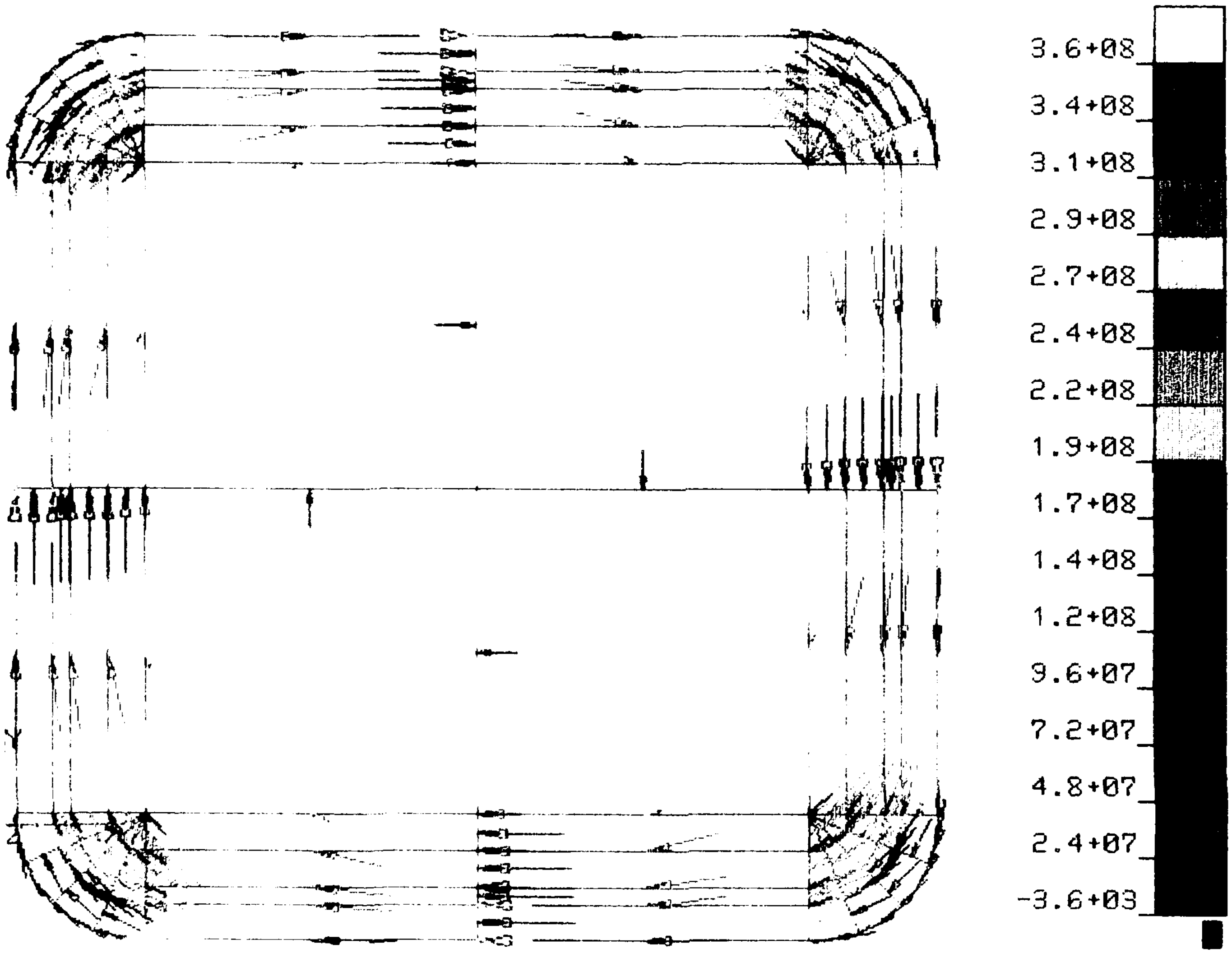


Figure 26. MFC: current density in upper face of copper plate in 20 μsec (vectorial representation)

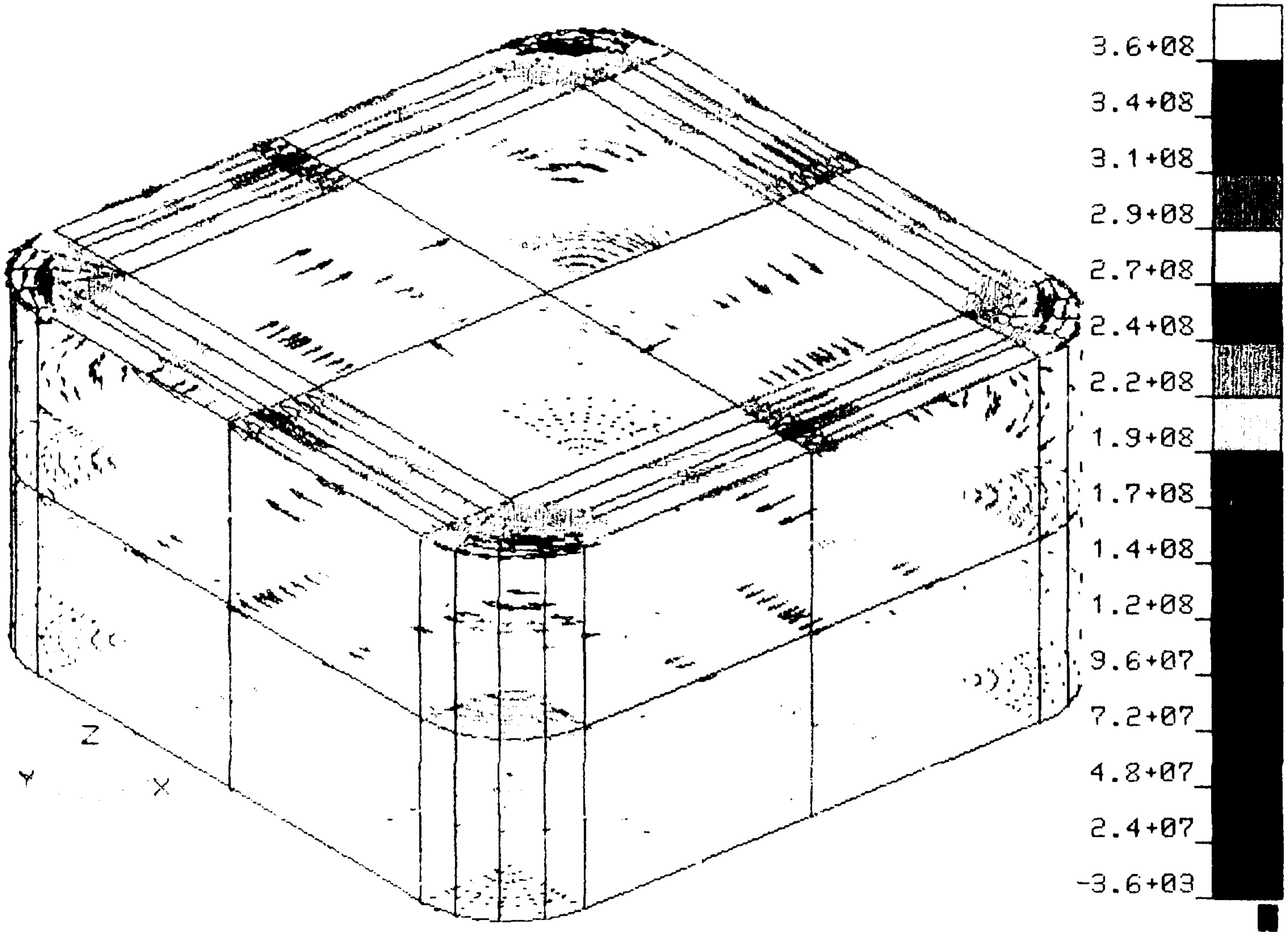


Figure 2   MFC: current density in copper plate at t = 20 μsec (3-D vectorial representation)

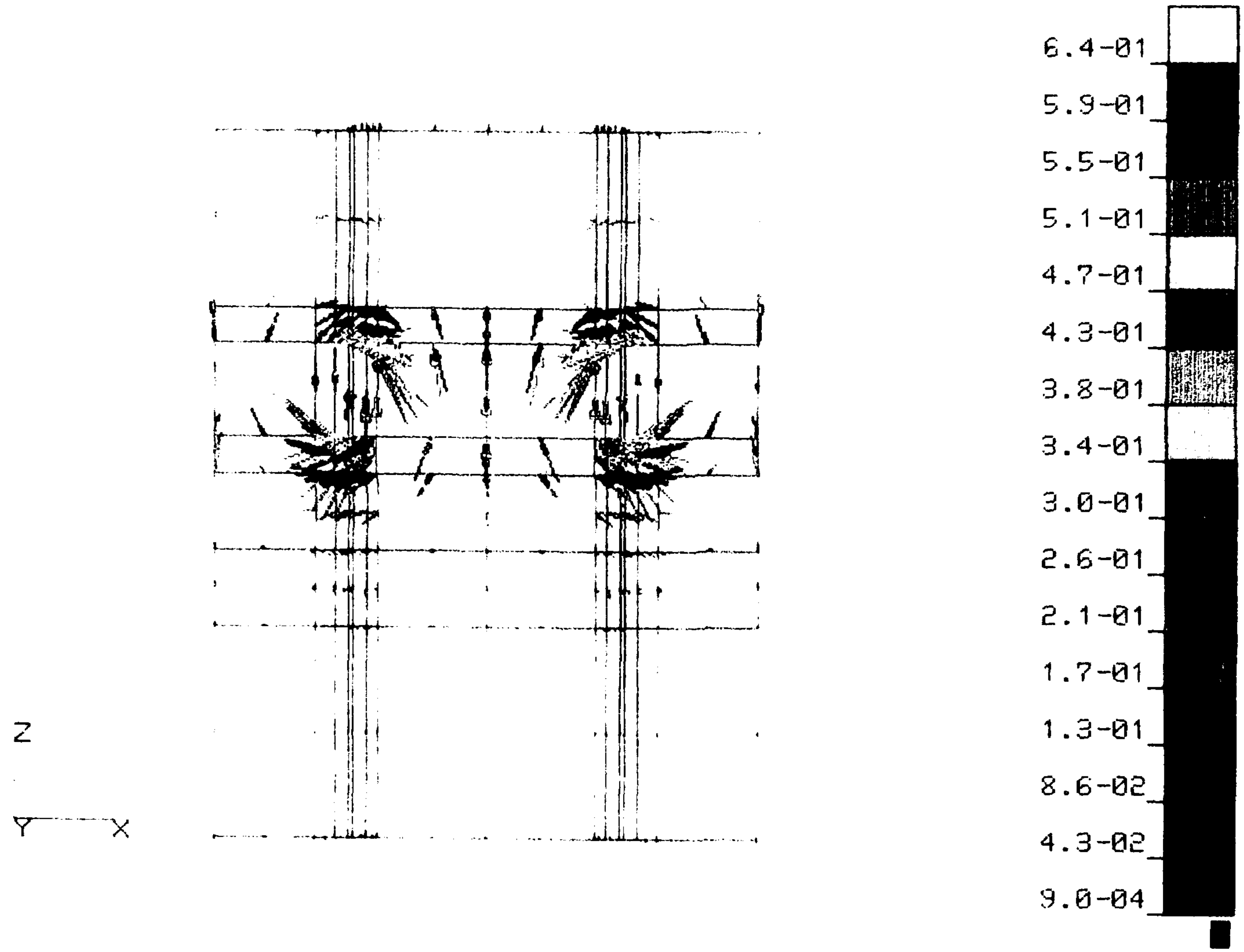


Figure 28. MFC: magnetic flux density at $t = 20\,\mu$sec (longitudinal section, vectorial representation)

CHAPTER 5

AN APPROACH OF CALCULATING FORCES IN ELECTROMECHANICAL ACCELERATORS BY USING THE METHOD OF FLUX DERIVATIVE

5.1 INTRODUCTION: POWER BALANCE AND THE MAGNETIC FLUX DERIVATIVE

The operation of electromagnetic macroparticle accelerators is based on the mechanism of conversion of electromagnetic into mechanical power[1]. Mechanical power is the rate at which mechanical work is performed by a distribution of forces acting on the moving projectile. In the case of electromagnetic launchers such forces are of electromagnetic origin. We will consider three ways of determining the mechanical forces of electromagnetic principals responsible for driving the electromagnetic accelerators:

a. As the distribution of body forces in the projectile volume and in the rest of the accelerator;

b As the global, resultant force (a generalized Lagrangian force, acting along a generalized coordinate);

c. And, finally, as stresses, using the artificial method of electromagnetic stress tensor, reducing the problem of electric and magnetic fields of force to that of an elastic continuum.

---

[1] Characteristic to electromagnetic macroparticle accelerators is their extremely high power ratings, for very short time intervals. The energies involved are relatively low with respect to other high energy industrial processes: 1 kwh=3.6 MJ When such energy is delivered in 2 msec, for example, it corresponds to a power of 1.8×10^9 w=1.8GW.

The theoretical tools which transform the expressions for forces from one form to another are related to:

a. The constancy of magnetic flux as the constraint used to assure the proper transformation in order to find the generalized Lagrangian forces along a generalized coordinate (global or integral characterization); and

b. The complex and subtle notion of flux derivative [25] as applied to electromagnetic systems in motion. Again, using the constancy of the magnetic flux as a constraint, we obtain all the equivalent body forces of electromagnetic origin (local or point characterization).

Since the electromechanical accelerators and their power supplies are based on magnetic flux compression devices, it is very significant that the constancy of the magnetic flux constraint in the Lagrangian formulation, and the same in the flux derivative approach, is the analytical tool which unifies the theoretical treatment of such devices.

Macroscopically, forces of electromagnetic origin can be expressed directly and elegantly since the electromagnetic component of the energy of a thermodynamic system can be distinctly[2] separated from other types of energy. The instantaneous power balance for an electromagnetic system can be written as:

[2] Electromagnetic energy of a system is that part of the internal energy which depends only on the state variables of the electromagnetic field, varying when they vary and remaining constant when they remain constant.

$$\oint_{s_v}\oint(\bar{E} \times \bar{H}) \bullet d\bar{s} = \iiint_v \bar{E}\bar{J} d\,v + \iiint_v \left(\bar{E} \bullet \frac{\partial \bar{D}}{\partial t} + \bar{H} \bullet \frac{\partial \bar{B}}{\partial t}\right) d\,v \tag{5.1}$$

In certain conditions of media homogeneity the balance can be written as:

$$\oint_{s_v}\oint(\bar{E} \times \bar{H}) \bullet d\bar{s} = \iiint_v \bar{E}\bar{J} d\,v + \frac{d}{dt}\iiint_v \left(\frac{D^2}{2\varepsilon} + \frac{B^2}{2\mu}\right) d\,v \tag{5.1a}$$

or

$$\oint_{s_v}\oint(\bar{E} \times \bar{H}) \bullet d\bar{s} = \iiint_v \bar{E}\bar{J} d\,v + \frac{d}{dt}\iiint_v (w_e + w_m) \; d\,v$$

where

$$w_e = \frac{\bar{D}\bar{E}}{2} = \frac{\varepsilon E^2}{2} = \frac{D^2}{2\varepsilon} \tag{5.1b}$$

$$w_m = \frac{\bar{B}\bar{H}}{2} = \frac{\mu H_2}{2} = \frac{B^2}{2\mu} \tag{5.1c}$$

are the electric and magnetic energy densities.

Or, in other words, the input of energy in unit time in the system (of volume V and surrounded by the surface S_v) expressed by the flux of Poynting's vector through the area S_v enclosing the system is spent as joule losses $(\bar{E} \bullet \bar{J})$ and also for increasing the electric energy density $\left(\frac{ED}{2}\right)$ and magnetic energy density $\left(\frac{HB}{2}\right)$ of the system.

As it is well known, electromagnetic power balance (5.1) is obtained directly from Maxwell's equations, after few mathematical manipulations, starting from:

$$\nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t} \qquad \text{(magnetic circuit law)} \tag{5.2}$$

$$\nabla \times \bar{E} = J + \frac{\partial \bar{B}}{\partial t} \qquad \text{(Faraday's law)} \tag{5.3}$$

and by dot multiplying by $\bar{E}$ both sides of (5.1A) and using the vectorial identity:

$$\nabla \bullet (\bar{E} \times \bar{H}) = \bar{H} \bullet \nabla \times \bar{E} - \bar{E} \bullet \nabla \times \bar{H}$$

we obtain:

$$-\nabla \bullet (\bar{E} \times \bar{H}) + \bar{H} \bullet \nabla \times \bar{E} = \bar{E} \bullet \bar{J} + \bar{E}\frac{\partial \bar{D}}{\partial t}$$

Substituting the value of $\nabla \times E$ from Faraday's law (2A) and regrouping the terms:

$$-\nabla \bullet (\bar{E} \times \bar{H}) = E \bullet J + \bar{E} \bullet \frac{\partial \bar{D}}{\partial t} + \bar{H} \bullet \frac{\partial \bar{B}}{\partial t}$$

By integrating over the volume of the system and transforming the volume integral on the left side into a closed surface integral covering the volume (using Gauss' divergence theorem), the relation (5.1) is obtained.

There is a subtle difficulty in using (5.1) to the case when the accelerated body (projectile) is moving. We assume that the surface of integration is driven together with the projectile, also ε and μ remain

"substantially" constant (or in other words, a point in movement maintains its properties[3]). Then the relation (5.1) remains valid with the assumption that the partial derivatives of $\bar{D}$ and $\bar{B}$ with respect to time, are replaced by their flux derivatives with respect to time

$$\frac{\partial D}{\partial t} \rightarrow \frac{d_f \bar{D}}{dt} \tag{5.4}$$

$$\frac{\partial \bar{B}}{\partial t} \rightarrow \frac{d_f \bar{B}}{dt} \tag{5.5}$$

where, the flux derivatives of the fields $\bar{D}$ and $\bar{B}$ are [25]

$$\frac{d_f \bar{D}}{dt} = \frac{\partial \bar{D}}{\partial t} + \bar{u} \quad div \quad \bar{D} + \quad curl \quad (\bar{D} \times \bar{u}) \tag{5.6}$$

$$\frac{d_f \bar{B}}{dt} = \frac{\partial \bar{B}}{\partial t} + \bar{u} \quad div \quad \bar{B} + curl \quad (\bar{B} \times \bar{u}) \tag{5.7}$$

and using the other two Maxwell's equations:

$$\text{div } \bar{D} = \rho_v \tag{5.8}$$

and

$$\text{div } \bar{B} = 0 \tag{5.9}$$

$$\frac{d_f \bar{D}}{dt} = \frac{\partial \bar{D}}{\partial t} + \rho_v \bar{u} + \nabla \times (\bar{D} \times \bar{u}) \tag{5.10}$$

---

[3] If μ remains "substantially" constant $\frac{ds\mu}{dt} = \frac{\partial \mu}{\partial t} + \bar{u} grad \mu$ where $\bar{u}$ is the instantaneous velocity of the point.

$$\frac{d_f \bar{B}}{dt} = \frac{\partial \bar{B}}{\partial t} + \nabla \times (\bar{B} \times \bar{u}) \tag{5.11}$$

5.2 GLOBAL (GENERALIZED) FORCES OF ELECTROMAGNETIC ORIGIN

One important way in which mechanical forces of electromagnetic origin responsible for driving electromagnetic accelerators can be evaluated is by employing the methods of analytical (Lagrangian) dynamics consistent with the first principle of thermodynamics and with Maxwell's equations.

For elementary displacements between two states of an insulated thermodynamic system, the first principle postulates that the sum of the elementary mechanical work performed by mechanical forces dL and of the elementary increase in the thermal energy dQ must equal the change in the internal energy of the system, $d\,W$.

$$-d\,W = dL + dQ \tag{5.12}$$

In the presence of the electromagnetic field, the terms dependent on state variables of the electromagnetic field can be separated from the internal energy of the system. Repeating the statement from the last section, the electromagnetic energy of a system is that part of the internal energy which varies with the variation of the electromagnetic state variables, remaining constant when they remain constant.

As a consequence, the balance of energy (5.12) can be written separately for the electromagnetic energy in which case the elementary mechanical work

is produced by mechanical forces of electromagnetic origin and the thermal energy accounts for the joule (and hysteresis) losses in the system.[4]

In rewriting the energy balance, we are using the fundamental identity for electromagnetic energy (A1). Then, for the infinitesimal time dt :

$$-d\,W = P_J dt + P_s dt + dL \tag{5.12a}$$

where

$$P_J = \iiint_v \bar{E} \bullet \bar{J} dv$$

represents the joule losses in the system (W)

$$P_s = -\oint_s\oint_v (\bar{E} x \bar{H}) \bullet \bar{ds}$$

represents the electromagnetic power input, flowing into the system through its boundary surface S_v (the external normal to the surface is by convention the positive one).

The chosen transformation - in the sense of Lagrangian dynamics - must permit an expedient evaluation of the mechanical work dL and from it, of mechanical forces of electromagnetic origin. Such a transformation is defined by the constraint:

$$P_j dt + P_s dt = 0 \tag{5.12b}$$

then

---

[4] Since the electromechanical systems are notoriously efficient, a lossless model of the system gives sufficient accuracy.

$$-d\,W\Big|_{P_s+P_{s_v}=0} = dL \tag{5.12c}$$

Let us express the constraint (5.12b) by using the identity (5.1):

$$P_f + P_{S_v} = -\iiint_v \left(\bar{E}\frac{d_f\bar{D}}{dt} + \bar{H}\frac{d_f\bar{D}}{dt} \right) dv = 0 \tag{5.12d}$$

In order to impose the constraint the integrand in the left side of the equation (5.12d) must be zero. The two instances in which this happens are (a) the fields (electric and magnetic) are zero, instance with no significance and (b) the flux derivatives are kept zero during the transformation (interesting!).

$\frac{d_f\bar{D}}{dt} = 0$ leads to the constraint $\Psi_{el.,_v} = const.$ (5.13a)

$\frac{d_f B}{dt} = 0$ leads to the constraint $\Phi_{m,_v} = const.$ (5.13b)

Substituting (5.13a and b) in (5.12c)

$$-\,d\,W\Big|_{\substack{\Psi_{el}=const.\\ \Phi_m=const.}} = dL \tag{5.12e}$$

actually splitting the energy dW in its electric and magnetic counterparts:

$$dL = -\,d\,W_e\Big|_{\Psi_{el}=const.} - dW_m\Big|_{\Phi_m=const.} \tag{5.12f}$$

For an electromechanical system with n degrees of freedom

$$dL = \sum_{k=1}^{h} F_k d\,x_k \tag{5.14}$$

where F_k, x_k are the generalized Lagrangian force and generalized Lagrangian coordinates, respectively.

By varying only the x_j coordinate:

$$dx_j \neq 0 \quad dx_k = 0 \quad (k \neq j)$$

$$F_j = -\left(\frac{\partial W_{el.}}{\partial x_j}\right)_{\psi_e = const} - \left(\frac{\partial W_m}{\partial x_j}\right)_{\Phi_m = const} \tag{5.15}$$

In summary, the steps followed in order to find the global, generalized Lagrangian force acting along the j th generalized coordinate are:

a. The system is constrained to change such as $[P_j + P_{v_e}] = 0$;

b. Among all possible virtual movements, the variation of only one generalized coordinate is chosen;

c. The expression found for the force is then valid independently of the variation of fluxes or currents (The constraints of <u>constant electric and magnetic flux densities</u> describe the transformation employed in order to derive the expression for the force and does not affect the application of this expression in the situation of variable flux, for example.)

d. Choosing the right generalized coordinates (their number being fixed at the number of degrees of freedom of the system) can provide a special insight into the electromagnetic accelerator operation.

Some pairs, generalized coordinate - generalized force are shown in table A.I.

TABLE A.I.

Generalized Coordinate	Generalized Force
Displacement, dx (m)	Force, F (N)
Rotation angle d α (rad)	Torque T (N•m)
Surface *ds* s (m^2)	Superficial Tension t(N/m)
Volume dv (m^3)	Pressure ρ (N/m^2)

For example, by expressing the energy as a function of the volume, the generalized force results as the (magnetic) pressure on the accelerator system.

Expressing the magnetic energy as functions of electric flux and magnetic flux (actually, magnetic flux linkage) as the independent variable may not be the most convenient choice. Many times the use of current as independent variable in the energy expression represents a more natural and convenient alternative. The Legendre transformation from analytical mechanics is the tool changing an energy function of a given set of variables into a new energy function of a new set of variables, the old and the new variables being related to each other by a joint transformation. The transformation has the remarkable property that is entirely symmetrical on both systems (the same transformation that leads from the old to the new system leads back from the new to the old system). Since, in this treatment,

only magnetic acceleration will be considered, the new energy function is the magnetic coenergy W'_m introduced through Legendre's transformation:

$$W'(I, x_1, x_2, \dots x_n) = \Phi_m I - W(\Phi_m, x_1, x_2, \dots x_n) \tag{5.16}$$

Then, the energy balance becomes:

$$dL = dW' \tag{5.17}$$

and the generalized force along the x_j coordinate:

$$F_j = +\left(\frac{\partial W'(I, x_1, x_2, \dots x_n)}{\partial x_j}\right)_{I=const} \tag{5.18}$$

5.3 LOCAL (BODY) FORCES OF ELECTROMAGNETIC ORIGIN AND THE FLUX DERIVATIVE METHOD

Electromagnetic accelerators and, especially, the projectile operate at extreme stress levels. For the same global force, a design can fail or successfully survive depending on how judicious the body forces are distributed.

The same methodology which led to finding the generalized force, starting from Maxwell's equations and being consistent with the first principle, can be also used to find the electromagnetic body forces.

A transformation, in the sense of Lagrangian dynamics, defined by the constraint (5.12b) was chosen in the form of equation (5.12e):

$$-dW\Big|_{\substack{\Psi_e=const\\ \Phi_m=const}} = dL \tag{5.12e}$$

In terms of power (differential energy dW in incremental time dt):

$$-\left(\frac{dW}{dt}\right)_{\substack{\Psi_e = const.\\ \Phi_m = const.}} = \frac{dL}{dt} = \iiint_v \bar{f} \bullet \bar{u}\, dv \tag{5.19}$$

Where $\bar{f}$ is the body force $\left(\frac{N}{m^3}\right)$ in the volume dv element and $\bar{u}$ is the velocity of the element $\frac{\mathrm{m}}{\mathrm{sec}}$. The energy W (J) can be substituted by the volume integral of the energy density $w\left(\frac{J}{m^3}\right)$:

$$-\frac{d}{dt}\left[\iiint_v w\, dv\right]_{\substack{\Psi_e = const.\\ \Phi_m = const.}} = \iiint_v \bar{f} \bullet \bar{u}\, dv \tag{5.19a}$$

In order to find the expression for the body force (5.19a) must be transformed in a way to obtain the vectorial factor which multiplies the velocity $\bar{u}$ in the right hand member of the equation. Splitting the energy density into its electric and magnetic counterparts yields:

$$\iiint_v \bar{f} \bullet \bar{u}\, dv = -\iiint_v \left[\left(\frac{\partial w_e}{\partial t}\right)_{\frac{d\bar{D}}{dt}=0} \left(\frac{\partial w_m}{\partial t}\right)_{\frac{d\bar{B}}{dt}=0}\right] dv \tag{5.19b}$$

(We have a derivative of a volume integral on a domain with moving bodies. An assumption is made that through the surface which surrounds the volume of the integration domain bodies do not "enter" or "come out" - or, in other words, the surrounding surface is being "driven" by polarized or magnetized bodies.)

The energy densities in 5.19b are given by 5.lb and 5.lc. Taking the derivative by parts

$$\iiint \bar{f} \bullet \bar{u} d v = -\iiint_v \left[\bar{E}\left(\frac{\partial \bar{D}}{\partial t}\right)_{\frac{d_f \bar{D}}{dt}=0} - \frac{E^2}{2}\frac{\partial \varepsilon}{\partial t} + \bar{H}\left(\frac{\partial \bar{B}}{\partial t}\right)_{\frac{d_f \bar{B}}{dt}=0} - \frac{H}{2}\frac{\partial \mu}{\partial t}\right] d v$$

(5.19c)

Observation: Considering that the material properties ε and μ remain substantially constant, or in other words, a point in movement maintains its properties, yields:

$$\frac{d s \varepsilon}{d t} = \frac{\partial \varepsilon}{d t} + \bar{u} \bullet grad \varepsilon = 0 \quad or \quad \frac{\partial \varepsilon}{d t} = -\bar{u} \bullet \overline{grad} \varepsilon ; \quad \frac{\partial \mu}{\partial t} = -\bar{u} \bullet \overline{grad} \mu$$

(5.12)

The condition of constant flux (electric and magnetic) imposes the constraint of zero flux derivatives for $\bar{D}$ and $\bar{B}$:

$$\frac{d_f \bar{D}}{d t} = \frac{\partial \bar{D}}{\partial t} + \bar{u} \quad div \quad \bar{D} + curl(\bar{D} \times \bar{u}) = 0 \tag{5.6}$$

and

$$\frac{d_f \bar{B}}{d t} = \frac{\partial \bar{B}}{\partial t} + \bar{u} \quad div \quad \bar{B} + curl(\bar{B} + \bar{u}) = 0 \tag{5.7}$$

and, by using Maxwell's equations:

$$div \quad D = \rho_0 \quad and \quad div \quad \bar{B} = 0;$$

$$\left(\frac{\partial \bar{D}}{\partial t}\right)_{\frac{d_f D}{dt}=0} = -\left[curl(\bar{D} \times \bar{u}) + \rho \bar{u}\right] \tag{5.6a}$$

$$\left(\frac{\partial \bar{B}}{\partial t}\right)_{\frac{df\bar{B}}{dt}=0} = -[curl(\bar{B} \times \bar{u})]$$

(5.7a)

Substituting (5.6a) and (5.7a) and (5.19d) back into (5.19c):

$$\iiint_v \bar{f} \bullet \bar{u} \quad dv = \iiint_v \quad [\rho_e \bar{E} \bullet \bar{u} + \bar{E} \bullet curl(\bar{D} \times \bar{u}) -$$

$$\left(\frac{E^2}{2} grad\varepsilon\right) \bullet \bar{u} + \bar{H} \bullet curl(\bar{B} \times \bar{u}) - \left(\frac{H^2}{2} grad\mu\right) \bullet \bar{u}\Big] dv \qquad (5.20a)$$

In the right side, two terms still do not have $\bar{u}$ as an explicit vectorial factor. In order to transform them we use the vectorial identities:

$$div[\bar{E} \times (\bar{D} \times \bar{u})] = \bar{E} \bullet curl(\bar{D} \times \bar{u}) - (\bar{D} \times \bar{u}) \bullet curl \quad E \qquad (5.20b)$$

$$div[\bar{H} \times (\bar{B} \times \bar{u})] = \bar{H} \bullet curl(\bar{D} \times \bar{u}) - (\bar{B} \times \bar{u}) \bullet curl \quad \bar{H} \qquad (5.20c)$$

Also, in the condition of constant electric and magnetic flux, the Faraday's law and the magnetic circuit law become

$$curl \quad \bar{E} = -\frac{d_f \bar{B}}{dt} = 0; \quad curl \quad \bar{H} = \bar{J} + \frac{df\bar{D}}{dt} = J \qquad (5.20d \text{ and } e)$$

The substitution of equations (5.20a, b, c, d, and e) into (5.20-a) leads to a remarkable expression:

$$\iiint_v \bar{f} \bullet \bar{u} dv = \iiint \left[\rho \bar{E} - \frac{E^2}{2} grad\varepsilon + (J \times B) - \frac{H^2}{2} grad\mu\right] \bullet \bar{u} dv$$

$$-\oint\!\!\oint_{s_e} [\bar{E} \times (\bar{D} \times \bar{n}) + H \times (\bar{B} \times \bar{n})] \bar{u} ds$$

(5.20f)

The surface integral, which results from the two divergence terms (5.20b) and (5.20c) becomes zero after applying Gauss' theorem since we chose the surface surrounding the domain such that, $\bar{u} = 0$ on it. Then, the body force $\bar{f}$:

$$\bar{f} = \bar{f}_e + f_m = \rho_f \bar{E} - \frac{E^2}{2} \bullet grad\varepsilon + \bar{J} \times \bar{B} - \frac{H^2}{2} \bullet grad\mu \tag{5.21}$$

where the electric field intensity $\bar{E}$ $\left(\frac{V}{m}\right)$ and the magnetic flux density $B\ (T)$ can be seen as volume force densities exerted by the field on the unit element of free charge ρ_f $\frac{C}{m^3}$ and respectively of moving charge, $\bar{J}$ (conduction current density $\frac{A}{m^2}$.)

The other two terms proportional to the gradients of electric permittivity and magnetic permeability represent the body forces exerted by the electric field on the polarizable materials and respectively by the magnetic field on magnetizable materials.

In summary, the steps followed in order to find the body force (the electromagnetic force density per unit volume) are:

a) The system is constrained to change such as the electric and magnetic flux remain constant during the transformation consistent with the first principle of thermodynamics and Maxwell's equations;

b) The constancy of the electric and magnetic flux is imposed by equating to zero the flux derivatives of the field vectors $\bar{D}$ and $\bar{B}$;

c) The body force is obtained as a vectorial factor, multiplying the local velocity $\bar{u}$ in the expression for electromagnetic power;

d) The body forces, thus obtained are universally valid for macroscopic, nonrelativistic electrodynamics;

3) In the above derivation, the variation of ε and μ with respect to the volume density of the materials (τ) has been neglected. If such variation exists, the electro- and, respectively, the magneto-striction terms will appear in (5.21).

$$\bar{F}_{es} = g\bar{r}\bar{a}d\left(\frac{E^2}{2}\tau\frac{\partial\varepsilon}{\partial\tau}\right); \bar{F}_{ms} = g\bar{r}\bar{a}d\left(\frac{H^2}{2}\tau\frac{\partial\mu}{\partial\tau}\right) \tag{5.22}$$

5.4 THE ELECTROMAGNETIC STRESS TENSOR AND ELECTROMAGNETIC IMPULSE

The Maxwell's electromagnetic stress tensor was actually based on Faraday's idea of an intermediary medium with special electric properties for transmitting the force. Let us again assume the domain V , surrounded by the surface S_v (Figure 4) The force $F_{21} = -F_{12}$ is introduced to maintain the equilibrium when the imaginary elastic strings are cut. This way, the global electromagnetic force $F\ (N)$ is equivalent to the stresses $\bar{T}_n\left(\frac{N}{m^2}\right)$ integrated

over the area S_v, which in turn, balances the body forces of electromagnetic origin $f\left(\frac{N}{m^3}\right)$ integrated over the volume V

$$\bar{F} = \iiint_v \bar{f} d v = \oint\oint \bar{T}_n d s \tag{5.23}$$

The replacement of body forces by stresses (using the Gauss' transformation of a volume integral into a surface integral) is performed for the particular case of a stationary regime $\frac{\partial \bar{D}}{\partial t} = 0$ and absence of material motion $\bar{u} = 0$.[5] At the end of this paragraph, the nonstationary regime will be considered while evaluating the electromagnetic impulse (so necessary in calculating different aspects of the recoil in electromagnetic guns).

Starting from equation (5.21) we will try to express the body force of electromagnetic origin as a divergence, in order to find an equivalent stress distribution

$$f = \rho_v \bar{E} - \frac{E^2}{2} grad\varepsilon + \bar{J} \times \bar{B} - \frac{H^2}{2} grad\mu =$$

$$\bar{E} div \bar{D} - grad\left(\frac{\varepsilon E^2}{2}\right) + \varepsilon grad \frac{E^2}{2} + curl\bar{H} \times \bar{B}$$

$$-grad\left(\frac{\mu H^2}{2}\right) + \mu grad \frac{H^2}{2} + curl\bar{E} \times \bar{D} + \bar{H} div \bar{B} \tag{5.21b}$$

in which the last two terms, added for symmetry, have zero value.

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[5] The expression for the body force (A-21) was obtained in the previous paragraph under the constraint of zero flux derivatives which in case of absence of motion $\bar{u} = 0$ yields the equations $curl\bar{H} = J$ and $curl\bar{E} = 0$.

By using the vectorial identity:

$$(\nabla \times \bar{E}) \times \bar{D} =$$

$$-\varepsilon\bar{E} \times (\nabla \times \bar{E}) = -\varepsilon\left[\nabla\left(\frac{E^2}{2}\right) - (\bar{E}\,\nabla)\bar{E}\right] \qquad (5.24a)$$

and

$$(\nabla \times \bar{H}) \times \bar{B} =$$

$$-\mu\bar{H} \times (\nabla \times \bar{B}) = -\mu\left[\nabla\left(\frac{H^2}{2}\right) - (\bar{H}\,\nabla)\bar{H}\right] \qquad (5.24b)$$

the volume force density expression becomes:

$$\bar{f} = \bar{f}_{el.} + \bar{f}_{mag.} =$$

$$\bar{E}div\bar{D} - grad\left(\frac{\varepsilon E^2}{2}\right) + \varepsilon grad\left(\frac{E^2}{2}\right) - \varepsilon grad\left(\frac{E^2}{2}\right) + (\bar{D}grad)\bar{E}$$

$$+ \bar{H}div\bar{B} - grad\left(\frac{\mu H^2}{2}\right) + \mu grad\left(\frac{H^2}{2}\right) - \mu grad\left(\frac{H^2}{2}\right) + (\bar{B}grad)\bar{H} \qquad (5.21c)$$

In relation (5.21c) there is a perfect symmetry between the electric and magnetic field quantities. The right side of the equation (5.21c) <u>expresses the tensoral divergence of a dydic:</u>

The x components of the electric and magnetic body forces in this relation are:

$$f_{elx} = E_x div\bar{D} + \bar{D} \bullet gradE_x - \frac{\partial}{\partial x}\left(\frac{\varepsilon E^2}{2}\right) = div(\bar{E}_x \times \bar{D}) - div\left(\bar{i}\,\frac{\varepsilon E^2}{2}\right)$$

(5.21d)

$$f_{mag\,x} = H_x div\bar{B} + \bar{B} \bullet gradHx - \frac{\partial}{\partial x}\left(\frac{\mu H\ 2}{2}\right) = div(H_x \times \bar{B}) - div\left(\bar{i}\ \frac{\mu H^2}{2}\right)$$

(5.21e)

considering also the y and z components, we can express, finally, the body force as a tensorial divergence:

$$\bar{f} = f_{el.} + f_{mag.} = \bar{i}\quad div\quad T_x + \bar{j}\quad div\quad \bar{T}_y + \bar{k}\quad div\quad \bar{T}_z \tag{5.25}$$

where $\bar{i}, \bar{j}, \bar{k}$ are the unit vectors of the three axes of coordinates x, y, z

and

$$\bar{T}_x = \left[E_x\bar{D} + H_x\bar{B} - \bar{i}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)\right] \tag{5.25a}$$

$$\bar{T}_y = \left[E_y\bar{D} + H_y\bar{B} - \bar{j}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)\right] \tag{5.25b}$$

$$\bar{T}_z = \left[E_z\bar{D} + H_z\bar{B} - \bar{k}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)\right] \tag{5.25c}$$

integrating by components

$$F = \iiint_v \bar{f}dv = \iiint_v \bar{i}\quad div\quad \bar{T}_x dv + \iiint_v \bar{j}\quad div\quad \bar{T}_y dv + \iiint_v \bar{k}\quad div\quad \bar{T}_z dv$$

(5.25d)

and using Gauss' theorem we can transform the volume integral of a divergence into a flux through a closed surface integral:

$$\oint\!\!\oint_{s_v}[\bar{i}\,(\bar{T}_x \bullet \bar{n}) + \bar{j}\,(\bar{T}_y \bullet \bar{n}) + \bar{k}\,(\bar{T}_z \bullet \bar{n})]ds = \oint\!\!\oint_{s_v}\bar{T}_n ds \tag{5.26}$$

where

$$\bar{\bar{T}} = \begin{pmatrix} T_{xx} T_{xy} T_{xz} \\ T_{yx} T_{yy} T_{yz} \\ T_{zx} T_{zy} T_{zz} \end{pmatrix}$$

is the electromagnetic stress tensor

and

$$\bar{T}_n = \bar{\bar{T}} \bullet \bar{n} = \left(\bar{\imath}\,; \bar{T}_x + \bar{\jmath}\,; \bar{T}_y + \bar{k}\,; \bar{T}_z\right) \bullet \bar{n}$$

$$F = \oint\!\!\oint_{s_v} \bar{\bar{T}}\ \bar{n}\quad ds = \iiint_v div\ \ \bar{\bar{T}}\ dv = \iiint_v \bar{f}\quad dv \tag{5.23a}$$

Then, the normal component of the stress tensor is:

$$\bar{T}_n = \bar{E}\left(\bar{D} \bullet \bar{n}\right) + \bar{H}\left(\bar{B} \bullet \bar{n}\right) - \bar{n}(w_{el.} + w_m) \tag{5.27}$$

and, the expression for the Maxwell's tensor:

$$\bar{\bar{T}} = \begin{pmatrix} E_x D_x + H_x B_x - w & E_x D_y + H_x B_y & E_x D_z + H_x B_z \\ E_y D_x + H_y B_x & E_y D_y + H_y B_y - w & E_y D_z + H_y B_z \\ E_z D_x + H_z B_x & E_z D_y + H_z B_y & E_z D_z + H_z B_z - w \end{pmatrix}$$

where $w = w_{el.} + w_m$ is the electromagnetic energy density $\left(\frac{J}{m^3}\right)$. Since we are interested only in the magnetic terms

$$\bar{f}_m = \bar{J} \times \bar{B} - \frac{H^2}{2} grad\mu = div\ \ \bar{\bar{T}}_m \tag{5.27a}$$

then

$$\bar{\bar{T}}_m \bullet \bar{n} = \bar{H}(\bar{B} \bullet \bar{n}) - \bar{n}\left(\frac{\bar{H} \bullet \bar{B}}{2}\right) \tag{5.27b}$$

where $\bar{n}$ is the positive normal to the surface. As a rule, $\bar{H}$ the intensity of the magnetic field bisects the angle between $\bar{n}$ and $\bar{T}$

Electromagnetic Impulse

Suppose that the regime is not stationary as was assumed at the beginning of the paragraph (5.20e). Then $curl\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$. In this case, the equation (5.23) must be modified:

$$\iiint_v \bar{f} d v - \oint\!\!\oint_{s_v} \bar{T}_n d s = -\frac{d}{dt}\iiint_v (\bar{D} \times \bar{B}) d v \tag{5.29}$$

or

$$\bar{F} + \frac{d}{dt}\iiint_v (\bar{D} \times \bar{B}) d v = \oint\!\!\oint_{s_v} (-\bar{\bar{T}})(-\bar{n}) d s \tag{5.29a}$$

If we consider an isolated system, which extends to infinity, then the surface integral disappears since $\bar{T}_n$ goes to zero faster than ds is increasing for an $R \rightarrow \infty$. This yields:

$$\bar{F} + \frac{d}{dt}\iiint_v (\bar{D} \times \bar{B}) d v = 0 \tag{5.29b}$$

The resultant force in an isolated mechanical system interacting with an electromagnetic system is zero; expressed in terms of the mechanical and electromagnetic[6] impulse:

$$\frac{d}{dt}\left[G_{mech.} + \iiint_v (\bar{D} \times \bar{B}) d v\right] = 0 \tag{5.30}$$

or the sum of the impulse is constant

$$\left[G_{mech.} + \iiint_v (\bar{D} \times \bar{B}) d v\right] = const.$$

In general,

$$\frac{d}{dt}\left(G_{mech.} + G_{el.mag.}\right) = \oint\!\!\oint_s \left(-\bar{\bar{T}}\right)(-\bar{n}) ds \; ^{8} \tag{5.30a}$$

The expression (5.30a) is important in evaluating the design questions related to recoil in electromagnetic guns. The volume integral of the electromagnetic force density, the Maxwell's stress tensor, and the electromagnetic impulse become tools in defining and calculating globally and locally the recoil in electromagnetic accelerators.

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[6] The density of electromagnetic impulse is $\overline{D} \times \overline{B} = \varepsilon\mu\overline{E} \times \overline{H} = \varepsilon\mu\overline{S}$

CHAPTER 6

SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK

6.1 SUMMARY

It is considered, in this work, that one of the reasons why the electromagnetic hypervelocity launchers and associated power supply technologies have not reached their expectations is the fact that most of the activity in this domain has been focused on simple railgun accelerators in conjunction with the questionable choice for homopolar generators known for their inherent low voltage and low power density. In order to increase the power densities, the homopolar machines have to be used in conjunction with heavy and voluminous inductors in which the magnetic energy is transiently stored and explosive opening switches which actually provide the high voltages necessary for driving high currents into railguns.

In order to drastically improve the performance of hypervelocity electromagnetic launchers the solution is to use as power supplies complex rotating flux compressors (beyond so-called compulsators) in conjunction with heteropolar electromagnetic launchers with variable structures.

The computation effort for the design of such complex magnetic flux compressors and heteropolar hypervelocity accelerators is tremendous in order to determine the time evolution of complex interacting electromagnetic fields.

For the demanding analysis and design of flux compression structures, a three-dimensional, finite element method formulation for the electromagnetic field diffusion is presented.

The formulation is based on the Galerkin technique since in the domain of parabolic partial derivative equations no variation principle is possible. This formulation requires large computational efforts since it solves the transient problem for all three components of the vector potential (as compared with axisymmetric f.e.m. codes solving for one component only).

One of the original features of the formulation is the introduction of superficial currents (sheets of currents) with linear current density, $\overline{K}$, flowing through the faces of each volume element (brick, prism, etc.) and compensating for the discrete nature of the finite element method and for other nonuniformities and discrepancies such as in prescribing the forcing functions (current densities, for example) element by element enforcing point-wise the solenoidality of flux density, B, current density, $\overline{J}$, and magnetic vector potential, $\overline{A}(div\overline{B} = 0;\ div\overline{J} = 0;\ div\overline{A} = 0)$.

The superficial currents with linear current densities $\overline{K}$ are introduced in the f.e.m. formulation as Lagrange multipliers, solenoidality of the three vectors, $\overline{B}$, $\overline{J}$, and $\overline{A}$, being the constraint imposed by the Lagrange multipliers.

The unfolding in time of the flux compression phenomena occurring in a three-dimensional flux compression structure (pulsed coil in front of

consecutively, producing remarkable velocities. Armature reaction will introduce, however, complex three-dimensional field distributions.

In the domain of the power supplies, complex rotary flux compressors as those proposed by Driga and Fair in [35] must be analyzed by the analytical and numerical methods treated in the present work.

One of the features desired to be incorporated into the f.e.m. 3-D code is the relative motion between the rotor and stator for the flux compressors and between the barrel and the projectile for the accelerators. In order to accommodate such problems, the f.e.m. computer code must be adaptive with possibility of continuous change in e structure of formulation.

The program must have an "initial" mesh changes according to the movement of the projectile.

The algorithm must assume two rigid bodies, both generating magnetic fields moving with relation to one another in an accelerated or decelerated manner.

A mesh of brick elements is generated around each bo 'y and those meshes meet along a mesh interface, Σ. To provide for a smooth sliding of one element with respect to another, a special five mode interface element is used [36]. The associated shape functions, $\psi_j(\zeta, \eta)$, are defined on a master element, Ω. The moving local coordinates on a typical element are then given by an isoparametric map.

While codes with adaptive mesh exist for solving problems in other domains of technology (such as supersonic flow in domains with moving bodies), they do not exist for electromechanical systems where they will have an important impact on analyzing highly transient systems.

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